Math Problem:
Compiled by Yukun Fung
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1. Find all integers n such that $2^n - 1$ is divisible by n.
2. Let $f(x) = \sin(x) + \sin(x^0)$, with domain the real numbers. Is f a periodic function? sin(x) is the sine of a real number, x, or equivalently, the sine of x radians, while $\sin(x^0)$ is the sine of x degrees. (hint: considering the equation $f'(x) = f'(0)$)
3. You are given n > 0 of each of the standard denomination US coins: 1¢, 5¢, 10¢, 25¢, 50¢, $1. What is the smallest n such that it is impossible to select n coins that make exactly a dollar?
4. Solve the equation $\sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-x}}}} = x$ and show the prove if you can. (All square roots are to be taken as positive.)
5. Let x be a real number. Which is greater, $\sin(\cos x)$ or $\cos(\sin x)$?
6. Let n be a positive integer, and let $S_n = \{n^2 + 1, n^2 + 2, \ldots, (n + 1)^2\}$. Find the cardinality of the set of pairwise products of distinct elements of S_n.

   For example, $S_2 = \{5, 6, 7, 8, 9\}$,

   $5 \times 6 = 6 \times 5 = 30$,
   $5 \times 7 = 7 \times 5 = 35$,
   $5 \times 8 = 8 \times 5 = 40$,
   $5 \times 9 = 9 \times 5 = 45$,
   $6 \times 7 = 7 \times 6 = 42$,
   $6 \times 8 = 8 \times 6 = 48$,
   $6 \times 9 = 9 \times 6 = 54$,
   $7 \times 8 = 8 \times 7 = 56$,
   $7 \times 9 = 9 \times 7 = 63$,
   $8 \times 9 = 9 \times 8 = 72$, and the required cardinality is 10
7. Let x, y, n be positive integers, with n > 1. How many solutions are there to the equation $x^n - y^n = 2^{100}$?
8. Show that $1 + x + x^2/2! + x^3/3! + \ldots + x^{2n}/(2n)!$ is positive for all real values of x.