Categories:
Generalization of what we do with $\text{Set}$.
(We have $\text{Set}$ and functions between sets)

\[
\text{SET} \xrightarrow{\text{objects (dots)}} \text{morphism} \xrightarrow{\text{arrows}}
\]

We require composition of arrows subject to axioms.

Def: if $f: A \to B$ is a morphism, we call $A$ the domain of $f$ and $B$ the codomain.

If $\text{codomain}(f) = \text{domain}(g)$ we have $g f$.

\[
f \circ g
\]

- Associative: $(f g) h = f (g h)$
  (so no parentheses needed)

For each object $A$ we have the identity morphism $\text{Id}_A: A \to A$ such that for any morphism $f$ with $\text{domain}(f) = A$, $f \circ \text{Id}_A = f$

Dual: For any morphism $g$ with $\text{codomain}(g) = A$,

$\text{Id}_A \circ g = g$
Def \( f : A \to B \) is an isomorphism means there exists a morphism \( f^* : B \to A \) s.t.
\[
\begin{align*}
& \, \, \, \, \, f \circ f^* = \text{Id}_B \\
& \, \, \, \, \, f^* \circ f = \text{Id}_A
\end{align*}
\]

Concretely denoted by \( f^{-1} \).

Concrete categories

Objects: Sets with structure
Morphisms: Functions that preserve that structure.

Groups: Obj: Groups Mor: Group homs.
\[
f(x \cdot y) = f(x) \cdot f(y)
\]

Ex: Composition of homs is a hom. \((Gp)\)