Define $F: \text{SET} \to \text{SET}$

objects: $F(S) = \text{Hom}_{h,k}(Y,S)$

$= \{ f: Y \to S : f \circ h = f \circ k \}$

Note: $\text{Hom}_{h,k}(Y,S) \subseteq \text{Hom}(Y,S)$

... the word *subfunctor*

Morphisms: Given $\alpha: S \to S'$, for $f \in \text{Hom}_{h,k}(Y,S)$

define $F(\alpha)(f) = \alpha \circ f$

Prove that $F(\alpha) \in \text{Hom}_{h,k}(Y,S')$

Show that saying that $(\pi, C)$ is universal for $F$ is equivalent to

the universal property above.

Now let $C = \frac{Y}{\sim}$, where $\sim$ is the smallest equivalence relation s.t. $\forall x \in X \ h(x) \sim k(x)$

and let $\pi$ be the canonical projection.

Finally show $\pi$ satisfies the universal property using the universal property of quotients.