\[ f(x) \neq 0 \text{ for some } x \in \Omega, \text{ so each } f_i \text{ has only finitely many roots.} \]

Since the zero set \( \Omega \) is locally finite, for each \( x \in \Omega \) there is an \( \epsilon > 0 \) such that \( f_i \) vanishes outside \( U_i \) and \( \bigcap_{j \neq i} U_j \cap (x - \epsilon, x + \epsilon) = \emptyset \).

We want functions \( f_i : M \to \mathbb{R} \), \( f_i \geq 0 \).

**Eulerian Realizer:** \( M \)

Any \( x \in \Omega \) is in only finitely many \( U_i \) and each \( f_i \) forms a smooth manifold \( \text{Partitions of Unity} \)

\[ f \]
\[
\int g(x)\, dx = \int g(x) \cdot 1 \, dx = \int g(x) \cdot f'(x) \, dx
\]

(01 Fru Integrate)

are not smooth

Example: Let \( f(x) = x^2 \). This works, but is bad, since \( f'(x) \) is not constant.

\[
\int f(x) \, dx \leq \int f'(x) \, dx \\
\frac{f(x)}{f'(x)} \leq \frac{f(x)}{f'(x)}
\]

since \( f(x) \geq 1 \) when \( f(x) > 0 \), we are still ok. Because we can re-define

\[
\frac{f'(x)}{f(x)} = 1 + \theta
\]

Note: If we relax the condition \( \frac{f'(x)}{f(x)} = 1 \),
Note: 

\[ \lim_{x \to c} f(x) = L \]

Let 
\[ f(x) = \begin{cases} \frac{c}{x} & \text{if } x \neq 0 \\ L & \text{otherwise} \end{cases} \]

Example of a smooth function on \( \mathbb{R} \):

\[ \int_{\mathbb{R}} \varphi(x) f(x) \, dx = \int_{\mathbb{R}} \varphi(x) \left( \frac{c}{x} \right) \, dx = 2 \]
Theorem \( T : \exists \delta > 0 \text{ s.t.} \)

Let \( \delta \) be the corner bump.

Cover \( Y \) with overlapping intervals.

\[
\begin{align*}
\forall x \in Y, \quad & \exists \eta \in (x - \delta, x + \delta) \\
\therefore g \circ f & = \eta
\end{align*}
\]