Name: __________________________

Please show all work. Supply brief narration with your solutions and draw conclusions.

1. A researcher starts a bacterial culture in a petri dish. Three days later the colony is 5 million strong. The day after that it reaches 7 million. Assuming the growth is exponential, what will the size be on the fifth day?

2. The level of medication for a while varies according to $s(t) = 15 + t^2 - t$ where time $t$ is measured in days. Compute the derivative of $s$ with respect to $t$ using the definition of derivative. Find and illustrate on a graph:
   (a) Initial level and after 3 days.
   (b) The instantaneous rates of change at those two times.
   (c) The average rate of change during that period of time.
   (d) The equation of the tangent line at $t = 3$.

3. A population of wasps $x_t$ satisfies the recursion $x_{t+1} = 5\sqrt{x_t}$. Find fixed points of the recursion (equilibria) and do some cobwebbing on a graph or numerical experimentation to determine their stability (attracting vs. repelling). Describe what happens to the population in the long run, if $x_0 = 0$. Same, if $x_0 = 1$.

4. Find $y'$ where
   (a) $y = x^4 \ln x$
   (b) $y = \frac{x^3}{\sin(2x)}$
   (c) $y = xe^x$
   (d) $\sin(2x) + \exp(3y) = y^2$

5. The windpipe contracts during a cough from the rest radius $R$ to radius $r \leq R$. The speed of the exiting is $v = a(R - r)r^2$, where $a$ is a positive constant. What value of $r$ maximizes speed?

6. Evaluate the following limits. Justify your answers. If you use l'Hôpital’s rule, be sure to specify which case.
   (a) $\lim_{x \to \infty} \frac{x}{3x + 1}$
   (b) $\lim_{x \to 0} x^4 \sin \left( \frac{1}{x} \right)$
   (c) $\lim_{x \to 0} \frac{\sin(3x)}{5x}$
   (d) $\lim_{x \to 0^+} x^4 \ln x$

7. Find antiderivatives for the following functions.
   (a) $\sin(3x)$
   (b) $x \sin(3x^2)$
   (c) $x^2 \sin(3x)$
   (d) $\ln x$ (hint: by parts)

8. After an initial bolus injection of 22 mg of a drug, a patient is placed on a drip delivering 1.2 mg per hour. If the drug is cleared by the patient at a rate of $0.8 - 0.05t^3$ mg/h as a function of time, what is the amount of drug in the patient 4 hours later?

---

**Score Sheet**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>total (80)</th>
<th>%</th>
</tr>
</thead>
</table>

THE UNIVERSITY OF TEXAS AT SAN ANTONIO