Calculus III, MAT 2213  Final Exam, May 4, 1993  Instructor: D. Gokhman
Show all pertinent work, answers alone are not sufficient. Box the answers.

Name: ______________________________

1. (40 pts.) SEQUENCES AND SERIES. Determine whether each of the following sequences or series converges to a real number.

(a) \( \frac{(\ln(n))^5}{n^{\frac{2}{3}}} \)  
(b) \( \left( \frac{n}{n+1} \right)^{n^2} \)  
(c) \( \sum_{n=1}^{\infty} \frac{4n^5 + 4}{n^7 + 2n^3} \)  
(d) \( \sum_{n=1}^{\infty} \frac{n^\frac{3}{2} \cos(n\pi)}{(n^5 + 1)^{\frac{1}{4}}} \)

2. (20 pts.)  
(a) Evaluate \( \sum_{n=0}^{\infty} \frac{1}{2^n} + \frac{2}{3^n} \).
(b) Find the interval of convergence for the series \( \sum_{n=1}^{\infty} \frac{(-1)^{n-1}n^n}{n} x^n \).

3. (20 pts.) Find the Taylor series with center \( c \) for the following functions

(a) \( f(x) = xe^x \), \( c = 0 \)  
(b) \( f(x) = \frac{1}{x^2} \), \( c = 1 \)

4. (20 pts.) CURVES. Sketch the curves and find equations describing them.

(a) a circle passing through the origin with center \((-3, 4)\).
(b) a parabola with vertex \((5, -2)\) and focus \((5, 2)\).

5. (20 pts.) Suppose the position of a spider on the floor is given as a function of time \( t \) by \( x = t^2 + 2t \), \( y = t^3 + 3t \). At time \( t \) find the velocity components \( \frac{dx}{dt}, \frac{dy}{dt} \) and the slope \( \frac{dy}{dx} \). At time \( t = 0 \) find an equation for the tangent line.

6. (40 pts.) POLAR COORDINATES.

(a) Sketch the curve \( r = \cos(2\theta) \). Find the area enclosed by one petal.
(b) Convert to Cartesian coordinates and sketch \( r = 2 \sec(\theta + \frac{\pi}{4}) \).

7. (40 pts.) VECTORS.

(a) Find the distance between the plane \( x + 4y - z = -4 \) and \((4, 2, 3)\).
(b) Find an equation for the plane containing the points \((-1, 3, 2)\), \((2, 3, 0)\), \((-3, -1, 0)\).

(Hint: pick 2 vectors parallel to the plane and take the cross product).

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