Instructor: D. Gokhman

Name: 

1. (30 pts.) Let \( \mathbf{u} = 2 \mathbf{i} + \mathbf{k} \), \( \mathbf{v} = \mathbf{i} - 2 \mathbf{j} \), \( \mathbf{w} = -\mathbf{j} + 5 \mathbf{k} \). Calculate the following:
   
   (a) \( \mathbf{u} \cdot (\mathbf{v} + 3 \mathbf{w}) \)  
   (b) \( \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) \)  
   (c) \( (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} \)  
   
   (d) \( |v - w| \)  
   (e) \( \text{comp}_u \mathbf{w} \)  
   (f) \( \text{proj}_u \mathbf{w} \)

2. (20 pts.) True/false questions. No explanation required.

   T F (a) There are exactly two unit vectors perpendicular to a plane in \( \mathbb{R}^3 \).
   T F (b) If \( \mathbf{u} \times \mathbf{v} = 0 \), then \( u = 0 \) or \( v = 0 \) or \( u \) is perpendicular to \( v \).
   T F (c) The line \( \mathbf{r} (t) = t(-\mathbf{i} - \mathbf{j} - \mathbf{k}) \) is perpendicular to the plane \( x + y + z = -2 \).
   T F (d) The line \( \mathbf{r} (t) = t \mathbf{j} \) lies in the plane \( x + z = 0 \).

3. (40 pts.) Let \( A = 2 \mathbf{i} + \mathbf{k} \), \( B = -\mathbf{i} + 2 \mathbf{j} \), \( C = \mathbf{j} - 3 \mathbf{k} \)

   (a) Find a parametric formula for the line through \( A \) and \( B \).
   (b) Find an equation for the plane through \( C \) perpendicular to the line.
   (c) Find the distance from \( A \) to the plane.
   (d) Find the distance from \( C \) to the line.

4. (40 pts.) Consider the plane curve \( \mathbf{r} (t) = t^2 \mathbf{i} - e^{t^5} \mathbf{j} \).

   (a) Find \( r'(t) \) and \( r''(t) \).
   (b) Find \( r(1) \), \( r'(1) \) and \( r''(1) \).
   (c) Find a parametric formula for the line tangent to \( \mathbf{r} (t) \) at the point \( r(1) \).
   (d) Find a parametric formula for the line perpendicular to \( \mathbf{r} (t) \) at the point \( r(1) \).

5. (30 pts.) Consider the circle of radius 2 centered at \( P = \mathbf{i} - \mathbf{j} \).

   (a) Find a parametric formula for the circle.
   (b) Find all \( x \) intercepts of the circle.
   (c) Pick one of the intercepts and find a parametric formula for the line tangent to the circle at that point.
6. (20 pts.) Compute the limits of the following functions as $(x, y) \to (0, 0)$:

   (a) $x^2 + y^2$  
   (b) $x^2y^2$  
   (c) $\frac{xy}{x^2 + y^2}$  
   (d) $\frac{x^2y}{x^2 + y^2}$

7. (40 pts.) Let $f(x, y) = x^3 + y^3 + 2xy$.

   (a) Find all the first and second partial derivatives of $f$.
   (b) Find and classify all critical points of $f$.
   (c) Find the values of $f$ at all critical points.
   (d) Sketch the traces by the planes $x = 0$ and $x - y = 0$.

8. (20 pts.) Let $f(x, y) = x^2 + y^2$.

   (a) Find an equation for the plane tangent to the graph of $f$ at the point given by $x = 1$, $y = -1$.
   (b) Sketch the level curve going through this point.