Name: __________________________

Please show all work.

1. (10 pts.) Describe and sketch the general solution of the system of linear equations given by the augmented matrix

\[
\begin{bmatrix}
1 & 2 & 0 & 1 \\
0 & 0 & 1 & 2
\end{bmatrix}
\]

Is the solution a subspace of \( \mathbb{R}^3 \)? Explain.

2. (15 pts.) For each of the following matrices describe and sketch the column space. What is the rank of each matrix?

(a) \[
\begin{bmatrix}
2 & -2 \\
1 & 1
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
2 & 0 \\
-1 & 1 \\
0 & 2
\end{bmatrix}
\]

3. (15 pts.) For each of the matrices in the preceding problem consider the corresponding linear map \( T \). In each case, what are the dimensions of the kernel and the range of \( T \)? Is \( T \) 1-1? Onto? Explain.

4. (15 pts.) Find the standard matrix for each linear map \( T: \mathbb{R}^n \to \mathbb{R}^n \), where

(a) \( n = 2 \) and \( T \) is the rotation by \( \pi/2 \).

(b) \( n = 3 \) and \( T \) is the rotation by \( \pi \) with respect to the \( x_2 \)-axis.

(c) \( n = 3 \) and \( T \) is the reflection with respect to the plane \( x_3 = 0 \).

5. (10 pts.) For which \( \lambda \) is the sequence \[
\begin{bmatrix}
11 - \lambda \\
-6
\end{bmatrix}, \begin{bmatrix}
18 \\
-10 - \lambda
\end{bmatrix}
\]
not linearly independent?

6. (15 pts.) Suppose \( A, B, C \) are invertible \( n \times n \) matrices. Solve the following equations for an \( n \times n \) matrix \( X \). Simplify.

(a) \( AXA^{-1} = B \)

(b) \( ABX + A = C \)

(c) \( ABCXCBXA = I \)

7. (10 pts.) Let \( A = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 3 & 6 & 0 & 6 \\ 5 & 10 & 4 & 14 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \).

It can be checked that \( A \) is row equivalent to \( B \). Find bases for nul \( A \) and col \( A \).

8. (10 pts.) Find \([v]_{\mathcal{B}}\), where

(a) \( v = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \) in \( \mathbb{R}^2 \) and \( \mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\} \).

(b) \( v = 2 + 3t \) in \( P_1 \) and \( \mathcal{B} = \{2 + t, 2 - t\} \).

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