Name: ______________________________

Please show all work.

1. Prove that for any positive integer \(n\) the quantity \(2^n3^{2n} - 1\) is divisible by 17.

2. Construct a truth table for \((p \rightarrow q) \rightarrow (q \rightarrow p)\). Is it a tautology, contradiction, or neither?

3. Prove that irrational real numbers form a set. You may assume \(\mathbb{Z}\) and \(\mathbb{R}\) are sets.

4. Suppose \(f: \mathbb{R} \rightarrow \mathbb{R}\) is given by \(f(x) = \sin x\). Find the following.
   (a) \(f(\mathbb{R})\)
   (b) \(f^*\{1, -1\}\)
   (c) \(A \subseteq \mathbb{R}\) such that \(A \neq \emptyset \land f^*(A) = \emptyset\)

5. Let \(f: \mathbb{R}^2 \rightarrow \mathbb{R}\) be given by \(f([x, y]) = x - y\). Prove that \(f\) is surjective and not injective. Find a one-sided inverse for \(f\) (with proof). Show that it is not unique.

6. Let \(f: X \rightarrow Y\) be a function and for \(x, x' \in X\) define \(x \sim x' \iff f(x) = f(x')\). Prove \(\sim\) is an equivalence relation on \(X\). Describe the equivalence classes for the case when \(f\) is injective. Same for when \(f\) is constant. Same for \(f: \mathbb{R} \rightarrow \mathbb{R}\) given by \(f(x) = \sin x\).

7. Let \(S = \{x \in \mathbb{Q}: (\exists n \in \mathbb{N})[x = 3^{-n}]\}\). If they exist, what are \(\max S\) and \(\min S\)? For \(S\) as a subset of \(\mathbb{Q}\), same question for \(\sup S\) and \(\inf S\). Prove your assertions about \(\min\) and \(\inf\).

8. Prove that the partial order \(\subseteq\) on \(\mathbb{R}\) is a linear order, by showing that for any two Dedekind cuts \(D\) and \(D'\) we have \(D \subseteq D' \lor D' \subseteq D\).

9. Show that the intersection of a nonempty family of initial segments in \(\mathbb{Q}\) is an initial segment. Give a concrete example of a nonempty collection of Dedekind cuts, whose intersection is not a Dedekind cut.

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