1. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function and let $F : \mathbb{R} \to \mathbb{R}^2$ be the function defined by $F(x) = [x, f(x)]$. Prove that $F$ is injective, but not surjective. Find a one-sided inverse for $F$ (with proof). Show that it is not unique.

Note: The image of $F$ is known as the graph of $y = f(x)$.

2. Let $f : X \to Y$ be a function and for $x, x' \in X$ define $x \sim x' \iff f(x) = f(x')$. Prove $\sim$ is an equivalence relation on $X$. Describe the equivalence classes for the case when $f$ is injective. Same for when $f$ is constant. Same for $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$.

3. Prove that a nonempty finite linearly ordered set has a minimum.

Hint: Induction on the size of the set.

4. Let $S = \{x \in \mathbb{Q} : (\exists n \in \mathbb{Z})[x = 2^n]\}$. If they exist, what are max $S$ and min $S$? For $S$ as a subset of $\mathbb{Q}$, same question for sup $S$ and inf $S$. Prove your assertions about min and inf.

5. Show that a union of initial segments in $\mathbb{Q}$ is an initial segment. Give a concrete example of a collection of Dedekind cuts, whose union is not a Dedekind cut.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>total (50)</th>
<th>%</th>
</tr>
</thead>
</table>

Prelim. course grade: %