Complex Variables / MAT 3223.001
Midterm 1 / February 23, 1998 / Instructor: D. Gokhman

Name: ______________________________

Please show all work and box the answers, where appropriate.

1. (10 pts.) Sketch the following subsets of the complex plane.
   (a) \( \{ z : |z + i| = |z - 1 - i| \} \)
   (b) \( \{ z : |z + i| = 2 \} \)

2. (10 pts.) Let \( f(z) = \frac{z + i}{z - i} \). Find and sketch:
   (a) \( f^{-1}(\{ w : |w| = 1 \}) \)
   (b) \( f(\{ z : \text{Re} z = 0 \}) \)

3. (20 pts.)
   (a) Show that if \( z \) is a solution of \( z^n = a \) and \( z_0 \) is a solution of \( z^n = 1 \), then \( z_0 z \) is a solution of \( z^n = a \).
   (b) Find all solutions of \( z^3 = 1 \).
   (c) Find a solution of \( z^3 = 8i \). (Hint: Express 8i in polar form.)
   (d) Find all solutions of \( z^3 = 8i \) and express them in polar and cartesian form. (Hint: You may use (a–c).)
   (e) Check your answers to part (d).

4. (20 pts.) Find the Maclaurin series expansion of each of the following functions and determine its radius of convergence.
   (a) \( \frac{1}{1 + 2z} \)
   (b) \( \frac{1}{i + z} \)

5. (14 pts.) Mix’n’match.

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<thead>
<tr>
<th></th>
<th>(a) ( f(z) = z )</th>
<th>(i) translation</th>
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<tbody>
<tr>
<td></td>
<td>(b) ( f(z) = -z )</td>
<td>(ii) isotropic expansion/contraction</td>
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<td>(c) ( f(z) = \overline{z} )</td>
<td>(iii) rotation</td>
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<td>(d) ( f(z) = -\overline{z} )</td>
<td>(iv) identity</td>
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<td>(e) ( f(z) = e^{i\theta} z )</td>
<td>(v) reflection with respect to the x axis</td>
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<td>(f) ( f(z) = a + z )</td>
<td>(vi) reflection with respect to the y axis</td>
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<td>(g) ( f(z) = rz )</td>
<td>(vii) reflection with respect to the origin</td>
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