Complex Variables / MAT 3223.001
Midterm 2 / April 1, 1998 / Instructor: D. Gokhman

Name: ____________________________________________________________

Please show all work and box the answers, where appropriate.

1. (20 pts.) Find all values of

(a) \( \log(-1) \)    (b) \((-i)^i\)

2. (20 pts.) Find all branch points of \( f(z) = \sqrt{1 - z^4} \). What is the smallest number of branch cuts needed to make \( f \) single valued? Sketch the branch points and an example of branch cuts as above. What would be the radius of convergence of the power series expansion of \( f \) at \(-1\)?

3. (20 pts.) Find all points in the complex plane, where each of the following functions of \( z = x + iy \) is analytic? complex differentiable?

(a) \( y^2 - x^2 + 2ixy \)    (b) \( \frac{x - iy}{x^2 + y^2} \)

4. (20 pts.) Show that if \( f(z) \) is analytic and real for all \( z \), then \( f \) must be constant.

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