Please show all work and explain your answers.

1. Expand $z^5 e^{z^2}$ in a Taylor series at $z = 0$. What is the radius of convergence?

2. Expand $\frac{1}{z^2 - 4}$ in a Laurent series convergent in a punctured disc centered at $-2$.
   
   What is the annulus of convergence?

3. Find and classify all singularities of $\frac{z}{\sin z}$.

4. Suppose $f_n(z)$ is a sequence of entire functions which converges to $z$ uniformly on $\mathbb{C}$. Prove that there exists $n^*$ such that for all $n \geq n^*$ $f_n(z)$ is a polynomial of degree 1.