Please show all work.

1. Prove by induction \( \sum_{k=1}^{n} k^2 = \frac{n(n + 1)(2n + 1)}{6} \).

2. Solve the linear congruence \( 13x \equiv 2 \pmod{31} \).

3. Compute \( 3^{45} \) modulo 11 by repeated squaring and reduction. Show work.

4. For which natural numbers \( a \) and \( b \) does the equation \( (a, x) = b \) have a solution? Prove your assertion.

5. In the commutative ring \( \mathbb{Z}_4[i] \) which of \( 1 + i \) and \( 1 + 2i \) is a unit and which is a zero divisor? Explain. What is the order of the unit?