Please show all work.

1. Partition the symmetric group $S_3$ by left cosets of the cyclic subgroup $\langle (2,3) \rangle$. Do the same with right cosets.

2. Suppose $G, G'$ are commutative multiplicative groups and $\varphi : G \to G'$ is a surjective homomorphism. For $y$ in $G'$ express its fibre $\varphi^{-1}(y) = \{ x \in G : \varphi(x) = y \}$ as a coset of $\ker \varphi$.

3. Find the solution set for the system of congruences

$$35x \equiv 15 \mod 50$$
$$x \equiv -2 \mod 30$$

4. Use Euclid’s algorithm for the polynomial ring $\mathbb{R}[x]$ to find the greatest common divisor and the Bézout coefficients for $x^2 + 3x + 2$ and $x^4 + x^3 + 3x + 3$.

5. Suppose $a$ is a real number and $\varphi : \mathbb{R}[x] \to \mathbb{R}$ is the evaluation map $\varphi(p(x)) = p(a)$. Prove that $\varphi$ is a ring homomorphism. What are its kernel and image?