1. (20 pts.) Let \( P \) be the plane in \( \mathbb{R}^3 \) spanned by \( \mathbf{i} - 3\mathbf{k} \) and \( \mathbf{j} + 2\mathbf{k} \). Let \( p = \mathbf{i} - \mathbf{j} + 2\mathbf{k} \). Let \( L \) be the line through 0 and \( p \). Let \( Q \) be the plane containing \( p \) parallel to \( P \).

   (a) Express \( Q \) and \( L \) in parametric form.
   (b) Express \( Q \) as the locus of a linear equation.
   (c) Is \( L \) perpendicular to \( P \)? Explain.

2. (21 pts.) Sketch the following manifolds and express them in parametric form:

   (a) Straight line in \( \mathbb{R}^3 \) through \( \mathbf{k} \) in the direction \( \mathbf{i} + \mathbf{j} \).
   (b) The ray (half-line) in \( \mathbb{R}^3 \) from 0 in the direction \( -\mathbf{i} - \mathbf{k} \).
   (c) Straight line segment in \( \mathbb{R}^2 \) from \( \mathbf{i} \) to \( \mathbf{j} \).
   (d) The circle in \( \mathbb{R}^2 \) of radius 2 centered at \( 0 \).
   (e) The circle in \( \mathbb{R}^2 \) of radius 2 centered at \( \mathbf{i} + 3\mathbf{j} \).
   (f) Right half of the circle in (d).
   (g) Circle in \( \mathbb{R}^3 \) of radius 3 centered at \( \mathbf{j} \) parallel to the \( x-z \) plane.

3. (10 pts.) Let \( v = \mathbf{i} - \mathbf{j} \). Let \( f : \mathbb{R}^3 \rightarrow \mathbb{R} \) be defined by \( f(u) = \text{comp}_v(u) = u \cdot v / |v| \).

   (a) Find the values of \( f \) on the standard basis vectors of \( \mathbb{R}^3 \).
   (b) Is \( f \) is a linear map? Explain.

4. (10 pts.) Let \( g : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) be the rotation by \( \pi/2 \) with respect to the origin.

   (a) Find the matrix that represents \( g \) with respect to the standard basis.
   (b) Write down the formula for \( g \).

5. (extra credit) Sketch the following parametrized manifolds in \( \mathbb{R}^3 \).

   (a) \( \cos t \mathbf{i} + \sin t \mathbf{j} + t\mathbf{k} \), where \( 0 \leq t < \infty \).
   (b) \( \sin \varphi (\cos \theta \mathbf{i} + \sin \theta \mathbf{j}) + \cos \varphi \mathbf{k} \), where \( 0 \leq \varphi \leq \pi/2 \), \( -\pi < \theta \leq \pi \).
   (c) \( r(\cos \theta \mathbf{i} + \sin \theta \mathbf{j}) + z\mathbf{k} \), where \( 1 \leq r \leq 2 \), \( -\pi/2 \leq \theta \leq \pi/2 \), \( 0 \leq z \leq 3 \).