Name: __________________________

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Let \( \mathbf{r} = [x, y, z] \) and \( r = |\mathbf{r}| \). Express \( \nabla \cdot (r^n \mathbf{r}) \) in terms of \( r \).

2. Let \( \omega = x \, dx + y \, dy + z \, dz \) and \( \eta = (x^2 + yz) \, dy \, dz + (y^2 + zx) \, dz \, dx + (z^2 + xy) \, dx \, dy \).
   Compute \( d\eta \) and \( \omega \wedge \eta \).

3. Given a steady temperature distribution \( f(x, y) = x^y \), how quickly does the temperature change as you start moving from the point \([3, 2]\) towards \([2, 3]\) with speed 5?

4. Use cylindrical coordinates to parametrize the solid cone \( z^2 = x^2 + y^2, \ -1 \leq z \leq 0 \).
   Integrate \( (x^2 + y^2 - z^2) \, dx \, dy \, dz \) over this cone.

5. Either find a scalar potential for \([3x^2, z^2/y, 2z \ln y]\) or explain why it fails to exist.

6. Either find a vector potential for \([xy^2z, -y^3z, x^2y + y^2z^2]\) or explain why it fails to exist.

7. Verify the fundamental theorem \( \int_{\Omega} d\omega = \int_{\partial \Omega} \omega \) with \( \omega = xz \, dx + yz \, dy + (x^2 + y^2) \, dz \) and the surface \( \Omega \) given by \( x^2 + y^2 + 2z = 1, \ z \geq 0 \) oriented with the upward normal. Sketch.
   Hint: Parametrize \( \Omega \) using cylindrical coordinates.
   Extra credit: Who first discovered the special case of the fundamental theorem that applies here?

8. Let \( F \) be a smooth vector field on \( \mathbb{R}^3 \) such that the flux of \( F \) through the lateral surface of a cone of volume \( b \) is \( q \). If \( F \) has constant divergence \( c \), what is the flux of \( F \) through the base of the cone? Explain.