Name: 

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Use cylindrical coordinates to integrate \((x^2 + y^2 + 3z^2) \, dx \, dy \, dz\) over the solid \(x^2 + y^2 \leq 4, \ -2 \leq z \leq 1\).

2. Either find a scalar potential for \(F\) or explain why it fails to exist, where
   \[
   (a) \quad F = [y, -x, 0] \quad (b) \quad F = [x, y, z]
   \]

3. Either find a vector potential for \(F\) or explain why it fails to exist, where
   \[
   (a) \quad F = [2x, -y, -z] \quad (b) \quad F = [3x, -y, -z]
   \]

4. Use Green’s theorem to calculate the area under one arch of the cycloid \([x, y] = [t - \sin t, 1 - \cos t]\) pictured below.

   \[
   \begin{array}{c|c|c|c|c|c}
   t & 0 & 0.5 & 1 & 1.5 & 2 \\
   \hline
   x & 0 & 1 & 2 & 3 & 4 \\
   \hline
   y & 1 & 1.5 & 2 & 1.5 & 1 \\
   \hline
   \end{array}
   \]

   Hint: Find a 1-form \(\omega\) such that \(d\omega = dx \, dy\) and recall that \(\int_\partial \omega = \int_\Omega \omega\)

5. Find the flux of \([7x, 8y, 9z]\) through the unit sphere.
   
   Hint: Don’t do it directly.

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Prelim. course grade: %