Name: __________________________________________________________________________

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Sketch and label 5 level sets of \( f(x, y) = xy \), including one at level 0.

2. In each case determine whether the limit exists, and if so, find the limit.
   
   \[
   \text{(a) } \lim_{{(x, y) \to 0}} \frac{x^4 - y^4}{x^2 + y^2} \quad \text{(b) } \lim_{{(x, y) \to 0}} \frac{x^2 - y^2}{x^2 + y^2}
   \]

3. If a cucaracha crawls south at 1 cm/s, it notices an increase in temperature at the rate of 2°C/s. If it crawls east at 1 cm/s, the temperature increases by 4°C/s. What is the rate of change of temperature if the cucaracha crawls northeast at 2 cm/s?

4. Find the divergence and curl of \([y^2z, \exp(xyz), x^2y]\).

5. Let \( f = (1 + x^2 + y^2)^{-1} \). Compute the Hessian matrix for \( f \) and find the quadratic Taylor approximation to \( f \) at the origin.