1. Consider the differential equation \( x^2 y'' - xy' + (1 - x)y = 0 \).

   (a) Find and classify all singularities.

   (b) Construct and solve the indicial equation.

   (c) Use the method of Frobenius to find two linearly independent solutions. For each of the two series involved compute the first three nontrivial terms.

2. A thin rod with length \( L = 1 \) meter and diffusivity \( \beta = 5 \) has an initial temperature distribution \( 1 - \cos(\pi x) \) degrees Celsius for \( 0 \leq x \leq 1 \). Assume that the ends of the rod are held at constant temperatures (what are they?).

   (a) What is the temperature distribution for \( t > 0 \)?

   (b) What is the limit of your solution as \( t \to \infty \), i.e. what is the steady state temperature distribution?