University of Texas at San Antonio

Real Analysis I, MAT 4213
Exam N\textsuperscript{0}2, 11/21/91
Instructor: D. Gokhman

Name: __________________________

1. (20 pts.) Suppose \( f, g : \mathbb{R} \to \mathbb{R} \) are continuous functions. Show that the set \( \{x: f(x) = g(x)\} \) is closed in \( \mathbb{R} \).

2. (20 pts.) Find all cluster points of the sequence \( \{a_n\} \), where \( a_n = \cos(n\pi) + \cos\left(\frac{1}{n\pi}\right) \).

3. (20 pts.) Classify all functions \( f : \mathbb{R} \to \mathbb{R} \) which are continuous and such that \( f(\mathbb{R}) \subset \mathbb{Q} \). Justify your answer.

4. (20 pts.) In each of the following determine whether the limit exists. If so, find the limit and prove your assertion.
   
   (a) \( \lim_{x \to 0} \frac{1 - \cos(x)}{x} \),
   
   (b) \( \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) \).

5. (20 pts.) Suppose \( f : \mathbb{R} \to \mathbb{R} \).

   Let \( D = \left\{ \frac{f(y) - f(x)}{y - x} : x, y \in \mathbb{R}, x \neq y \right\} \). Show that if \( D \) is a bounded set, then \( f \) is uniformly continuous.