Throughout, suppose \((x_n)\) and \((y_n)\) are sequences of real numbers, \(x, y \in [-\infty, \infty]\), \(x\) is a partial limit of \((x_n)\) and \(y\) is a partial limit of \((y_n)\).

1. (30 pts.) Determine whether each of the following statements is true in general. If true, prove it. If false, give a specific counterexample.

(a) If \(\exists m \forall n \geq m \ x_n \geq 0\), then \(x \geq 0\).

(b) If \(\forall m \exists n \geq m \ x_n \geq 0\), then \(x \geq 0\).

(c) If \(\exists m \forall n \geq m \ y_n > 0\), then \(y > 0\).

(d) If \(0\) is a partial limit of \(x_n - y_n\), then \(x\) is a partial limit of \((y_n)\).

(e) If \(x_n - y_n \to 0\), then \(x\) is a partial limit of \((y_n)\).

(f) \(x + y\) is a partial limit of the sequence \((x_n + y_n)\).

2. (10 pts.) Suppose \(A \subseteq \mathbb{R}\) and \(a\) is a limit point of \(A\). Prove that there exists a sequence in \(A \setminus \{a\}\) that converges to \(a\).

3. (10 pts.) Let \(a = \lim \inf x_n\) and \(b = \lim \sup x_n\). Suppose \(U\) is an open interval containing the closed interval \([a, b]\). Prove that \(\exists m \forall n \geq m \ x_n \in U\).