Name: __________________________

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Suppose $G$ is a group such that $\forall a, b, c \in G \ ab = ca \Rightarrow b = c$. Prove that $G$ is abelian.

2. Show that in a finite group the number of all elements of order 3 is even.

3. Let $G = GL(n, Q)$ be the multiplicative group of invertible $n \times n$ matrices with rational coefficients and $H = SL(n, Q) = \{A \in G: \det A = 1\}$. Prove that $H$ is a subgroup of $G$. Prove or disprove that $H$ is normal in $G$.

4. Let $G$ and $H$ be as in the preceding problem. Suppose $A, B \in G$ and $\det A = \det B$. Prove that $A$ and $B$ belong to the same left coset of $H$.

5. Prove that for $n \geq 3$ the symmetric group $S_n$ has trivial center. What is $Z(S_2)$?

6. Let $A$ be the set of all elements of the ring $Z \oplus Z$ whose first coordinate is even. Prove that $A$ is an ideal. Is it maximal? Prove your assertion.

7. Suppose $\varphi: R \rightarrow S$ is a ring homomorphism from a ring with unity $R$ to an integral domain $S$ such that $\varphi(R) \neq \{0\}$. Prove that $\varphi(1) = 1$.

8. Prove that $x^p + x + 1$ and $2x + 1$ determine the same function $Z_p \rightarrow Z_p$. 

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