Name: 

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Let \( H = \{(), (12)(34), (13)(24), (14)(23)\} \). Prove that \( H \) is a subgroup of \( A_4 \). What is its index \( [A_4 : H] \)? Is \( H \) normal in \( A_4 \)? Prove your assertion.

2. Let \( A \) be the set of all polynomials in \( \mathbb{Z}[x] \) such that the constant coefficient is divisible by 3. Prove that \( A \) is an ideal of \( \mathbb{Z}[x] \). Is it maximal? Prove your assertion.

3. Let \( R \) be the ring of continuous functions \( \mathbb{R} \to \mathbb{R} \) with the usual pointwise operations. Is there a function in \( R \) that is neither a zero divisor nor a unit of \( R \)? Provide an explicit example or prove that no such example exists.

4. Suppose \( R \) is an integral domain. What is the largest and what is the smallest possible number of elements of \( R \) that are their own cubes? Explain.

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Prelim. course grade: %