Midterm 1 / 2009.3.5 / MAT 4233.001 / Modern abstract algebra

Name: ____________________________

Please show all work and justify your answers.

1. Prove that $a \in \mathbb{Z}_n$ has a multiplicative inverse if and only if $a$ is relatively prime to $n$. What is the multiplicative inverse of 3 in $\mathbb{Z}_{10}$?

2. Suppose $G$ is a group where each nontrivial element has order 2. Prove that $G$ is abelian.

3. Suppose $G$ is a cyclic group of order 18. How many subgroups does it have? Explain.

4. Suppose $G$ is a group with $|G|$ a positive integer power of 2. Prove that $G$ has an element of order 2.

5. Let $H = \{(), (12)(34), (13)(24), (14)(23)\}$. Prove that $H$ is a subgroup of $S_4$ (you may use the word *similarly* as appropriate). What is its index? Is $H$ isomorphic to $\mathbb{Z}_4$? Explain.