1. Suppose \( a \in \mathbb{Z}_n \). Prove \( a \in U(n) \) if and only if \( a \) is relatively prime to \( n \). What is \( |U(n)| \) if \( n \) is prime? Explain. What is the multiplicative inverse of 5 in \( \mathbb{Z}_{18} \)?

2. Prove or disprove \( U(8) \cong U(12) \).

3. Let \( H = \{(), (12)(34), (13)(24), (14)(23)\} \). Prove that \( H \) is a subgroup of \( A_4 \) (you may use the word similarly as appropriate). List all the cosets of \( H \) in \( A_4 \). Is \( H \) isomorphic to \( \mathbb{Z}_4 \)? Explain.

4. Suppose \( G \) is a group with \( |G| = 11 \). Prove or disprove that \( G \) must be cyclic.

5. Suppose \( G \) is a group with \( |G| \) a positive integer power of a prime \( p \). Prove that \( G \) has an element of order \( p \).