1. Show \[
\begin{bmatrix}
7 & 1 \\
4 & 5
\end{bmatrix}
\in GL(2, \mathbb{Z}_{11})
\] and find its inverse.

2. Determine the subgroup lattice of the symmetric group $S_3$. Pick a nontrivial subgroup of $S_3$ other than $A_3$ and find all its left cosets and right cosets.

3. Suppose $G$ is an abelian multiplicative group with $a, b \in G$. Prove by induction that $(ab)^n = a^n b^n$ for all $n \in \mathbb{Z}$. Find an example to show that the result may not hold if $G$ is not abelian.

4. Suppose a subgroup $H$ of $\mathbb{Z}$ contains two distinct primes. Prove that $H = \mathbb{Z}$.

5. Suppose $G$ is a multiplicative group and $a \in G$. Define $\varphi : \mathbb{Z} \rightarrow \langle a \rangle$ by $\varphi(k) = a^k$. Prove that $\varphi$ is an isomorphism if and only if $|a| = \infty$. 

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>total (50)</th>
<th>%</th>
</tr>
</thead>
</table>

Prelim. course grade: %