Name: ___________________________  Pseudonym: ___________________________

Please show all work. (Notation: $S^n = \{ u \in \mathbb{R}^{n+1}: |u| = 1 \}$, $D^n = \{ u \in \mathbb{R}^n: |u| \leq 1 \}$)

1. (15 pts.) Let $A = \{ (x, y) \in \mathbb{R}^2: x - y \geq 1 \}$. Define $f: \mathbb{R}^2 \to \mathbb{R}^2$ by

$$f(x, y) = \begin{cases} (x, y) & \text{if } (x, y) \in A \\ (0, 0) & \text{if } (x, y) \not\in A \end{cases}$$

(a) Sketch $A$.
(b) Describe or sketch the set of points of discontinuity of $f$.
(c) Illustrate the discontinuity of $f$ with a sketch.

2. (30 pts.) In each case construct an example or state that there can be no such example.

(a) A collection of closed subsets of the plane whose union is not closed.
(b) $A = f^{-1}(B)$, where $f$ is continuous, $A$ is not connected, and $B$ is connected.
(c) $A = f^{-1}(B)$, where $f$ is continuous, $A$ is not compact, and $B$ is compact.
(d) A continuous $f: D^2 \to S^1$ such that $f$ restricted to $S^1$ is the identity function.
(e) An open cover for a set $A$ without a finite subcover, where

(i) $A = \{ u \in \mathbb{R}^2: 1 < |u| \leq 2 \}$  
(ii) $A = \{ u \in \mathbb{R}^2: 1 \leq |u| \}$

3. (10 pts.) Are the following spaces connected? If not, construct a separation.

(a) $\{(x, y) \in \mathbb{R}^2: y = mx, \text{ where } m \in \mathbb{Q}\}$  
(b) $\{(x, y) \in \mathbb{R}^2: x + y \in \mathbb{Q}\}$

4. (15 pts.) If $X$ and $Y$ are homeomorphic, construct a homeomorphism $f: X \to Y$.
If $X$ and $Y$ are not homeomorphic, why not?

(a) $X = \mathbb{R}$, $Y = (0, 1)$.  
(b) $X = \mathbb{R}$, $Y = S^1$.  
(c) $X = S^1$, $Y = S^2$.

5. (15 pts.) For the following transformations of the plane $f$ compute the winding number of the image of the unit circle relative to the origin.

(a) $f(x, y) = (-x, y)$  
(b) $f(x, y) = (-x, -y)$  
(c) $f(x, y) = (x - 2, y)$

6. (5 pts.) Construct a homotopy in the plane from the unit circle to the origin.

7. (30 pts.) True/false — circle your choice. Justification is not required.

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Have a great break!  

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