1. (20 pts.) Suppose \( f : X \to Y \) is a function, \( A \subseteq X \), and \( B \subseteq Y \).
   (a) Prove that \( X \setminus f^{-1}(B) = f^{-1}(Y \setminus B) \).
   (b) Disprove by counterexample that \( Y \setminus f(A) = f(X \setminus A) \).

2. (20 pts.) Let \( A = \{(x, y) \in \mathbb{R}^2 : x > 1\} \) Define \( f : \mathbb{R}^2 \to \mathbb{R}^2 \) by
   \[
   f(x, y) = \begin{cases} 
   (x, y) & \text{if } (x, y) \in A \\
   (0, 0) & \text{if } (x, y) \not\in A
   \end{cases}
   \]
   (a) Sketch \( A \). Prove that \( A \) is open in \( \mathbb{R}^2 \).
   (b) Show that \( f \) is not continuous and illustrate with a sketch.

3. (25 pts.) In each case give an example or state that there can be no such example.
   (a) A collection of open subsets of \( \mathbb{R}^2 \) whose union is not open.
   (b) A collection of open subsets of \( \mathbb{R}^2 \) whose intersection is not open.
   (c) An open cover for a set \( A \) without a finite subcover, where
      (i) \( A = \{u \in \mathbb{R}^2 : 0 < |u| \leq 1\} \)
      (ii) \( A = \{u \in \mathbb{R}^2 : |u| \leq 1\} \)
      (iii) \( A = \{u \in \mathbb{R}^2 : |u| < 1\} \)

4. (20 pts.) True/false — circle your choice. Justification is not required.
   Throughout this problem \( f \) is a continuous function.
   T F (a) \( f(A) \subseteq B \Leftrightarrow A \subseteq f^{-1}(B) \).
   T F (b) If \( X \) is compact, then \( f(X) \) is compact.
   T F (c) If \( f : \mathbb{R} \to \mathbb{R} \), then there exists \( x \in \mathbb{R} \) such that \( f(x) = 0 \).
   T F (d) A subset \( A \) of \( \mathbb{R}^2 \) is compact \( \Leftrightarrow \) \( A \) is closed and bounded.
   T F (e) A subset \( A \) of \( \mathbb{R}^2 \) is closed \( \Leftrightarrow \) \( A \) is not open.

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