1. (10 pts.) ROOTS
Find all solutions of the equation $z^5 = -4 + 3i$ and plot them.

2. (10 pts.) STEREOGRAPHIC PROJECTION
Given two points $z_1, z_2 \in \mathbb{C}$, find the length of the chord between the corresponding points on the Riemann sphere? Pick two specific values for $z_1$ and $z_2$ and sketch. For which $z_1, z_2$ are the two corresponding points on the sphere diametrically opposite?

3. (10 pts.) EULER’S FORMULA
For $a, b \in \mathbb{R}$ find the partial sum: $\sum_{k=0}^{n} \sin(a + kb)$

4. (40 pts.) TRANSFORMATIONS
(a) Sketch the image of the segment $\text{Re } z = \text{Im } z$ between 0 and $1 + i$ under $w = e^z$? What is the arclength of the image?
(b) Show that the group of Möbius transformations is not commutative.
(c) For which $a \in \mathbb{R}$ is the group generated by $w = e^{ia}z$ finite?
(d) Find the group of transformations corresponding to rotations of the Riemann sphere with respect to the imaginary axis.

5. (40 pts.) INTEGRATION
(a) Find $\int z^n \log(z) \, dz$, where the path of integration is the unit circle (counterclockwise).
(b) Find $\int \frac{z^{2z}}{z^3} \, dz$, where the path of integration is the unit circle (counterclockwise).
(c) Suppose $f/z$ is continuous in a sector centered at the origin with aperture $\theta$. Let the path $\gamma(r)$ be the intersection of $|z| = r$ with the sector. Show that $\int_{\gamma(r)} \frac{f(z)}{z} \, dz \rightarrow \theta i f(0)$ as $r \rightarrow 0$.
(d) Suppose $g$ is entire and $z g(z) \rightarrow 0$ as $z \rightarrow \infty$. Show that integrals of $g(z) \, dz$ along any two rays from 0 to $\infty$ are equal, assuming they exist. Hint: use the results of part (c).
6. (30 pts.) POWER SERIES
Find the Maclaurin series for each of the following functions and determine its radius of convergence

(a) \( \frac{z}{z+2} \)
(b) \( \tan(z) \)
(c) \( \int_0^z \frac{\sin(z)}{z} \, dz \)

These problems are from *A collection of problems on complex analysis* by Volkovyskii, Lunts and Aramanovich, 1960 (Dover 0486669130, QA331.7.V6513 1991)