1. Prove that a continuous real-valued function on a topological space that is zero on a dense subset must be the zero function.

2. Given a family of topological spaces, pick a subset in each and prove that in general, the product of the subsets’ closures is the closure of their product.

3. Suppose $X$ is a topological space and $A \subseteq X$. Recall that $A$ is a retract of $X$ whenever there exists an onto continuous function $X \to A$ that is identity on $A$.
   
   (a) Prove that $A$ is a retract of $X$ if and only if a continuous function on $A$ can be extended to $X$.
   
   (b) Prove that if $X$ is Hausdorff, then $A$ must be closed in $X$.
   
   (c) Prove that the unit circle in the plane is a retract of the plane punctured at the origin.

4. Given a point in a discrete space, which filters converge to that point? What happens in a trivial space?

5. Prove that the intersection of compact subsets of a Hausdorff space is compact.