1. Let \( f(t) = t^4 - 2t^2 \). Find all the critical points of \( f \) on the interval \(-2 \leq x \leq 2\). Use the second derivative to determine concavity at the critical points. Find the global minimum and the global maximum of \( f \) on the interval. Where do they occur?

\[
\begin{align*}
\text{\texttt{> f:=t^4-2*t^2;}} & \\
\text{\texttt{> df:=diff(f,t); factor(\%);}} & \quad f'=4t^3-4t \\
\text{\texttt{> cp:=solve(df);}} & \quad df=12t^2-4 \\
\text{\texttt{> ddf:=diff(df,t);}} & \quad cp:=0,1,-1 \\
\text{\texttt{> [cp]:map(xx->subs(t=xx,ddf),\%);}} & \quad ddf=12t^2-4 \\
\text{\texttt{> [cp,-2,2]: map(xx->subs(t=xx,f),\%);}} & \quad [0,1,-1] \\
\text{\texttt{> plot(f,t=-2..2);}} & \quad [-4,8] \\
\end{align*}
\]

2. Find indefinite integrals of the following functions

(a) \( e^{2t}(1+e^{2t})^5 \)  
(b) \( t \cos(2t) \)

\[
\begin{align*}
\text{\texttt{> exp(2*t)*(1+exp(2*t))^5; int(\%);}} & \quad \int e^{2t}(1+e^{2t})^5 \, dt = \frac{1}{12}(1+e^{2t})^6 + C \\
\text{\texttt{> t*cos(2*t); int(\%);}} & \quad \int t \cos(2t) \, dt = \frac{1}{4}t \cos(2t) + \frac{1}{2} \sin(2t) + C \quad \text{Let } u = e^{2t}, \quad du = 2e^{2t} \, dt \\
\end{align*}
\]
3. Show that the improper integral \( \int_1^\infty \frac{1}{\sqrt{x^2 + x^2}} \, dx \) converges and find an upper bound.

\[
\sqrt{x} > 0 \ , \ \sqrt{x^2 + x^2} > x^2 \ , \ \frac{1}{\sqrt{x^2 + x^2}} < \frac{1}{x^2} \\
\int_1^\infty \frac{1}{\sqrt{x^2 + x^2}} \, dx \leq \int_1^\infty \frac{1}{x^2} \, dx = \int_1^\infty x^{-2} \, dx = \left[ -x^{-1} \right]_1^\infty = -1 + 1 = 0 + 1 = 1
\]

4. For the autonomous differential equation \( \frac{dx}{dt} = x - ax^2 \), where \( a \) is a positive constant, draw the phase-line diagram, find the equilibria, and determine their stability.

\[
\frac{dx}{dt} = x - ax^2 = x(1-ax) = 0 \quad \text{when} \quad x = 0 \quad \text{or} \quad x = \frac{1}{a} \quad \Rightarrow \quad \text{equilibria}
\]

\[
\begin{align*}
\frac{dx}{dt} & = x - ax^2 \\
\frac{dx}{dt} & = \frac{dx}{dt} = \sqrt{h(t)} \\
\int h^{-\frac{1}{2}} \, dh = -\frac{t}{2} + C \\
\frac{h^{\frac{1}{2}}}{v^2} = 2h
\end{align*}
\]

5. Solve the Torricelli equation \( \frac{dh}{dt} = -\sqrt{h} \) with initial condition \( h(0) = 1 \). When is \( h = 0 \)?

\[
> \text{eq:=diff}(h(t),t) = -\sqrt{h(t)}; \\
> \text{ic:=h(0)=1;} \\
> \text{dsolve}([\text{eq},\text{ic}],h(t)):\text{allvalues}(); \\
> \text{sol:=subs}(h(t),\text{solve}()); \\
> \text{plot}(\text{sol},t=0..2);
\]