1. Expand \((z - i)^{-1} + (z - 2)^{-1}\) in a Laurent series centered at the origin and valid in the annulus \(\{z : 1 < |z| < 2\}\).

\[
\frac{1}{z - i} + \frac{1}{z - 2} = \frac{1}{z} \frac{1}{1 - \frac{i}{z}} - \frac{1}{2} \frac{1}{1 - \frac{2}{z}}
\]

\[
= \frac{1}{2} \sum_{n=0}^{\infty} \frac{i^n}{2^n} - \frac{1}{2} \sum_{n=0}^{\infty} \frac{1}{2^n} z^n
\]

\[
= \sum_{n=0}^{\infty} \frac{i^n}{2^n} - \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} z^n
\]

\[
= \sum_{n=-\infty}^{\infty} \frac{-1}{2^n} z^n - \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} z^n
\]

2. Integrate \(\cot z\) around the unit circle.

\[
\cot z = \frac{\cos z}{\sin z} \text{ has singularities at } z = k\pi, \ k \in \mathbb{Z}
\]

Only \(z = 0\) is inside the unit circle.

\[
\cot z = \frac{1}{z} - \frac{z}{2} + \cdots
\]

\[
\therefore \text{Res} = 1, \text{ so } \int \cot z \, dz = 2\pi i
\]
3. Use Rouche’s theorem to determine the number of zeros, counted with multiplicity, of $z^3 - 5z + 1$ outside the unit disc.

\[ f + g = \text{deg } f = 2^3 + 1 \]
\[ f = \text{max deg } = 2^3 - 5z + 1 \]
\[ g = -\text{max } = 5z \]

\[ |f + g| = |z^3 + 1| \leq |z|^3 + 1 = 2 < 5 = |5z| = |g| \leq (f + |g|) \]

\[ \therefore \text{ Rouche applies, so } f \text{ has the same # of zeros inside the unit disc as } g, \text{ which is 1.} \]

\[ f \text{ is a cubic so has 3 zeros, so } f \text{ has} \]
\[ 2 \text{ zeros outside the unit disc.} \]

4. Find a fractional linear transformation that maps the unit disc to the right half-plane.

Pick 3 points on the unit circle counterclockwise:
\[ -i, \ i, \ i \text{ and map them to } 0, \ 1, \ \infty \]

Then \[ T(z) = \frac{z + i}{z - i} \]

\[ \frac{-i}{1 + i} = \frac{-i - i}{1 - i} \cdot \frac{-i}{z - z} \]

Take the unit disc to the upper half-plane.

Then rotate by $-\frac{\pi}{2}$, i.e. multiply by $e^{i \frac{-\pi}{2}} = \text{cis}(-\frac{\pi}{2})$

\[ \text{Ans: } (-i)(-i) \cdot \frac{z + i}{z - i} = \boxed{\frac{z + i}{z - i}} \]

(Case unique)