Simplices and boundary:

Boundary homomorphism $\partial : \Delta_k \to \Delta_{k-1}$ is defined on a $k$ simplex $\sigma$ by $\partial(\sigma) = \sum_{i=0}^{k} (-1)^i \sigma[v_0, \ldots, \hat{v}_i, \ldots, v_k]$

$\partial[v_0, v_1] = [v_1] - [v_0]$

$\partial[v_0, v_1, v_2] = [v_1, v_2] - [v_0, v_2] + [v_0, v_1]$

$\partial[v_0, v_1, v_2, v_3] = [v_1, v_2, v_3] - [v_0, v_2, v_3] + [v_0, v_1, v_3] + [v_0, v_1, v_2]$

Reference: