Linear algebra glossary

**Vector space:** A vector space $V$ is a nonempty set with addition and scalar multiplication. The operations must satisfy certain axioms for arbitrary vectors $u, v, w$ in $V$ and numbers $a, b$.

- **Closure:** $u + v$ and $au$ are in $V$
- **Commutative:** $u + v = v + u$
- **Associative:** $u + (v + w) = (u + v) + w$ and $(ab)u = a(bu)$
- **Zero:** there exists a zero vector $0$ such that $u + 0 = u$
- **Additive inverse:** there exists $-u$ such that $u + (-u) = 0$
- **Distributive:** $a(u + v) = au + av$ and $(a + b)u = au + bu$
- **Unitary:** $1u = u$

**Superposition principle I:** If $v_1, ... v_n$ are vectors in $V$, then any linear combination $c_1v_1 + ... + c_nv_n$ is in $V$.

**Subspace:** A subspace of $V$ is a nonempty subset $H$ of $V$ that is closed under addition and scalar multiplication.

**Span:** If $S$ is a subset of $V$, the span of $S$ is the set (in fact, subspace) of all linear combinations of vectors in $S$.

**Linear map:** A linear map is a function between vector spaces $T: V \to W$ that preserves addition and scalar multiplication. Specifically $T(u + v) = T(u) + T(v)$ and $T(au) = aT(u)$.

**Superposition principle II:** If $T$ is linear, then $T(c_1v_1 + ... + c_nv_n) = c_1T(v_1) + ... + c_nT(v_n)$

**Linear independence:** A sequence of vectors $v_1, ... v_n$ is linearly independent means that the homogeneous vector equation $c_1v_1 + ... + c_nv_n = 0$ has only the trivial solution $c_1 = ... = c_n = 0$.

**Basis:** A basis for $V$ is a linearly independent sequence of vectors $v_1, ... v_n$ that spans $V$.

**Dimension:** Dimension of $V$ is the (unique) number of elements in any basis for $V$.

**Coordinates:** If $v$ is a vector in $V$ and $B = \{b_1, ... b_n\}$ is a basis for $V$, the coordinate vector $[v]_B$ consists of the coefficients (weights) $c_1, ... c_n$ of the unique expansion $v = c_1b_1 + ... + c_nb_n$.