Complex Variables homework I

1.1

1. Compute the followings

   a) \((-2 + 3i) + (1 + i)\),  
   b) \((2 + i)(1 - 2i)\),  
   c) \(\text{Re}((1 - i)^3)\),  
   d) \(\text{Im}((2 + i)(1 + 3i)i)\).

2. Show that the solution to \(az = b\) is \(z = \frac{ib}{|a|^2}\).

3. Compute

   a) \(\frac{1}{1 + i}\),  
   b) \(\frac{2 - 3i}{1 - 2i}\),  
   c) \(\frac{(1 - i)^2}{(2 - i)^3}\).

4. Find the norms and principal arguments of the followings

   a) \(2 + \sqrt{3}i\),  
   b) \(\frac{i}{1 + 2i}\),  
   c) \((1 + i)^2(1 - i)^3\).

   Write down the polar form of the above numbers.

5. Find all values of

   \(\sqrt[3]{1 + i}\),  
   \(\sqrt[23]{2 + \sqrt{3}i}\),  
   \((1 - i)^{-2/3}\).

6. Solve \(z^{4/3} + 2i = 0\).

1.2

1. Draw the sets of the interior and boundary points of

   a) \(|z - i| > 0\),  
   b) \(\frac{1}{3} < \frac{1}{|z - i|} < \frac{1}{2}\).

2. Show that the disk \(\{z : |z - z_0| < r\}\) is open and connected.

3. Show that the rectangle \(\{x + yi : |x| < a, |y| < b\}\) is open and connected for any given \(a, b\).

4. Show that \(A \cup B\) is connected if \(A \cap B \neq \emptyset\) and \(A, B\) are connected.