Enter an ODE into Maple

```maple
> restart:
> with(DEtools):
> with(plottools):
```

Solving ODE with Maple

Using the command `DEplot` to solve this ODE. Notice how complicated the formula looks like!

```maple
> eqn := diff(y(t),t) = f(y(t),t);

\[ eqn := \frac{dy}{dt} = y(t)^2 + t^2 \]

Save the plot of the solution with initial condition \( y(0)=1 \) to a variable `trueplot` for later use.

```maple
> trueplot := DEplot(eqn, y(t), t=0..0.5, [[y(0)=1]]):
```

Euler method

```maple
> y := 1; t := 0; h := .1; P[0] := [point([t,y], color=blue)];

\[ y := 1 \]
\[ t := 0 \]
\[ h := 0.1 \]

\[ P_0 := \text{POINTS}([0, 1], \text{COLOUR}(RGB, 0., 0., 1.00000000)) ]

> for i from 1 by 1 to 5 do y := y + h*f(y,t): t := t+h: P[i] := point([t,y], color=blue): od:

> Euler_plot := seq(P[n], n=0..5);

\[ \text{Euler}\_\text{plot} := \text{POINTS}([0, 1], \text{COLOUR}(RGB, 0., 0., 1.00000000)), \text{POINTS}([0.1, 1.1], \text{COLOUR}(RGB, 0., 0., 1.00000000)), \text{POINTS}([0.2, 1.222], \text{COLOUR}(RGB, 0., 0., 1.00000000)), \text{POINTS}([0.3, 1.3753284], \text{COLOUR}(RGB, 0., 0., 1.00000000)), \text{POINTS}([0.4, 1.573481221], \text{COLOUR}(RGB, 0., 0., 1.00000000)), \text{POINTS}([0.5, 1.837065536], \text{COLOUR}(RGB, 0., 0., 1.00000000)) ]

> plots[display](Euler_plot);
```
> plots[display](Euler_plot, trueplot);
Improved Euler method

The code for improved Euler follows.

```plaintext
> y:=1; t:=0; h:=.1; P_I[0]:=[point([t,y], color=black, symbol=DIAMOND)];

    y := 1
    t := 0
    h := 0.1

    P_I_0 := [POINTS([0., 1.], COLOUR(RGB, 0., 0., 0.), SYMBOL(DIAMOND))]

> unassign('i');
> for i from 1 by 1 to 5 do z:=f(y,t): y:=y + (h/2)*(z+f(y+h*z, t)): t:=t+h: P_I[i]:=point([t,y], color=black, symbol=DIAMOND): od:
```
> Improved_Euler := seq(P_I[n], n = 0..5):
> plots[display](Euler_plot, Improved_Euler);

> plots[display](Euler_plot, Improved_Euler, trueplot);
\textbf{Improved Euler procedure}

We can make the above into a procedure so that it can be called for different sets of parameters. This procedure needs:
- \( F \): the function defines the right hand side of the eqn.
- \( y_0, t_0 \): the initial data
- \( h, \text{stop_num} \): the stepsize and the number of recorded points
- \( \text{clr}, \text{sym} \): the color and symbol for points.

The procedure will return a sequence of points ready to be plotted by \texttt{display} function in the plots package.

\begin{verbatim}
> IEuler:= proc(F, y0, t0, h, stop_num, clr, sym)
    local y, t, z, i, P;
    y:=y0: t:=t0:
    for i from 1 by 1 to stop_num
      do
        z:=F(y,t):
    end:

> IEuler:= proc(F, y0, t0, h, stop_num, clr, sym)
    local y, t, z, i, P;
    y:=y0: t:=t0:
    for i from 1 by 1 to stop_num
      do
        z:=F(y,t):
    end:

> IEuler:= proc(F, y0, t0, h, stop_num, clr, sym)
    local y, t, z, i, P;
    y:=y0: t:=t0:
    for i from 1 by 1 to stop_num
      do
        z:=F(y,t):
    end:
\end{verbatim}
\[ y := y + \frac{h}{2} \cdot (F(y + h \cdot z, t) + z) \]
\[ t := t + h \]
\[ P[i] := \text{point}([t, y], \text{color}=\text{clr}, \text{symbol}=\text{sym}) \]

\[ \text{end proc:} \]

This is how we use it.

```maple
> Improved_Euler := IEuler(f, 1, 0, .1, 5, RED, CIRCLE): plots [display](Improved_Euler);
```

>