Consider a nonlinear second order ode

\[ eqn := \frac{d^2}{dt^2} x(t) - \sin\left(\frac{d}{dt} x(t)\right) + (x(t))^2 = x(t) \]  

(1)

We rewrite this equation as a system of first order odes by putting \( y(t) = x'(t) \). We then set

\[ f := (x, y) \rightarrow y; \quad g := (x, y) \rightarrow x - x^2 + \sin(y); \]

\[ f := (x, y) \rightarrow y \]

\[ g := (x, y) \rightarrow x - x^2 + \sin(y) \]  

(2)

Solving for \( y'(t)=x''(t) \), we end up with the following system

\[ sys := \frac{d}{dt} x(t) = y(t), \quad \frac{d}{dt} y(t) = x(t) - x(t)^2 + \sin(y(t)) \]  

(3)

Plot the direction field around the steady states (0,0) and (1,0)
Zoom in at (0,0) to see that it is an Unstable Saddle point

> dfieldplot(sys, [x(t), y(t)], t = 0..10, x = -.2 ..0.2, y = -.2 ..0.2)
Zoom in at (0,1) to see that it is an Unstable Spiral point

\[ \text{dfieldplot}([\text{sys}], [x(t), y(t)], t = 0..10, x = .8..1.2, y = -.2..0.2) \]
Plot some trajectories (solution curves) starting near (0,0)

> `DEplot([sys], [x(t), y(t)], t = 0..200, [[x(0) = 0.04, y(0) = -0.02], [x(0) = 0.03, y(0) = -0.02], [x(0) = -0.04, y(0) = 0.025]], x = -0.05..0.05, y = -0.05..0.05, numpoints = 3000)`
Plot some trajectories (solution curves) starting near (1,0)

> \( \text{DEplot([sys, [x(t), y(t)], t = 0..200, [[x(0) = 0.995, y(0) = -0.02], [x(0) = 1.01, y(0) = 0.02], [x(0) = 1.001, y(0) = 0.025]], x = .9..1.1, y = -0.05..0.05, numpoints = 3000])} \)
Use Euler scheme to numerically solve the system with initial condition \((x(0),y(0))=(0.1,0.1)\). We record the x-coordinate (which is \(x(t)\))

\[
\begin{align*}
X &:= 0.1; Y := 0.1; s := 0; h := .05; numpoints := 10; P[0] := \text{point([s, X], color = blue)} : \\
\text{for } i \text{ from 1 by 1 to numpoints do } &X := X + h*f(X, Y); Y := Y + h*g(X, Y); s := s + h; \\
P[i] := \text{point([s, X], color = blue)} : \\
\text{od: Eulerplot} := \text{seq(P[n], n = 0..numpoints)} :
\end{align*}
\]

\[
\begin{align*}
X &:= 0.1 \\
Y &:= 0.1 \\
s &:= 0 \\
h &:= 0.05 \\
numpoints &:= 10 \\
\text{plots[display]}(\text{Eulerplot, scaling = constrained}); plot1 := %;
\end{align*}
\]
Use Euler scheme to numerically solve the system with initial condition \((x(0),y(0)) = (1.01,0.2)\) near the spiral point. We record the \((x,y)\)-coordinates.

```maple
> X := 1.01; Y := 0.02; s := 0; h := .05; numpoints := 40; P[0] := [point([X, Y], color = blue)]; for i from 1 by 1 to numpoints do X := X + h * f(X, Y); Y := Y + h * g(X, Y); s := s + h; P[i] := point([X, Y], color = blue); od: EulerplotXY := seq(P[n], n = 0..numpoints):
      
X := 1.01
Y := 0.02
s := 0
h := 0.05
numpoints := 40

> plots[display](EulerplotXY, scaling = constrained); plot1 := %;
```

\(\text{plot1} := \text{PLOT}(...)\)
Ask Maple to plot the trajectory.

\[ \text{plot1 := PLOT(...)} \quad (7) \]

\[ > \text{DEplot([sys], [x(t), y(t)], t = 0 .. 2, \{[x(0) = 1.01, y(0) = 0.02]\}, x = .99 .. 1.05, y = -0.02 .. 0.03); plot2 := %} \]
Compare with our approximation points.

\[ \text{plot2} := \text{PLOT}(...) \]