Elementary Differential Equation

In this worksheet, we will use the command \texttt{int} to solve some very elementary differential equations.

1 Example 1

Consider the equation

\[ \frac{dy}{dx} = \frac{x + \sin(2x)}{y^2} \]

We separate the variables (to make everything involving \(y\) (respectively, \(x\)) stays in one side of the equation). In our case, we simply multiply both sides by \(y^2\) and then by \(dx\). The equation now becomes

\[ y^2 \frac{dy}{dx} = (x + \sin(2x)) \]

Integrating both sides

\[ \int y^2 \, dy = \int (x + \sin(2x)) \, dx \]

Ask Maple to carry out the calculation (add an arbitrary constant to one side of the result)

\[ \int y^2 \, dy = \int (x + \sin(2x)) \, dx + C \]

We can then solve for the unknown \(y\)

\[ y = \left( \frac{1}{12} x^2 - \frac{1}{12} \cos(2x) + 24 C \right)^\frac{1}{2} \]

2 Example 2

Consider the equation

\[ \frac{dz}{dt} = -z^2 t (t^2 + 4)^4 \]

Maple gives us both real and complex \(y\). We are interested only in real valued function so that the solution is

\[ y = \left( \frac{1}{12} x^2 - \frac{1}{12} \cos(2x) + 24 C \right)^\frac{1}{2} \]
Given that \( z(0) = 1 \). That is, if \( t = 0 \) then \( z = 1 \)

Using Leibnitz notation
\[
\frac{dz}{dt} = -z^2 t (t^2 + 4)^4
\]

Separate the variables (\( z \) and \( t \))
\[
\frac{dz}{z^2} = -t (t^2 + 4)^4 \, dt
\]

Integrating both sides
\[
\int \frac{1}{z^2} \, dz = \int -t (t^2 + 4)^4 \, dt
\]

We get (for a general constant \( C \))
\[
-\frac{1}{z} = -\frac{1}{10} t^{10} - 2 t^8 - 16 t^6 - 64 t^4 - 128 t^2 + C
\]

Using the condition \( z = 1 \) when \( t = 0 \). We get the equation to determine the constant \( C \).
\[
\text{subs}(\{(z=1, t=0),\%\});
\]

The equation now becomes
\[
-\frac{1}{z} = -\frac{1}{10} t^{10} - 2 t^8 - 16 t^6 - 64 t^4 - 128 t^2 - 1
\]

Solve for \( z \), we get the solution as
\[
\text{solve}(-1/z = -1/10*t^{10}-2*t^8-16*t^6-64*t^4-128*t^2-1, z);
\]

\[
\frac{1}{10} t^{10} + 20 t^8 + 160 t^6 + 640 t^4 + 1280 t^2 + 10
\]