SECTION 2.1  Basics of Functions and Their Graphs

**Objectives**

1. Find the domain and range of a relation.
2. Determine whether a relation is a function.
3. Determine whether an equation represents a function.
4. Evaluate a function.
5. Graph functions by plotting points.
6. Use the vertical line test to identify functions.
7. Obtain information about a function from its graph.
8. Identify the domain and range of a function from its graph.
9. Identify intercepts from a function’s graph.

A relation is a function when each input has exactly one output.

**Vertical Line Test**

graph \( y = f(x) \)

\[ f(x) = 3x + 1 \]

\((1, 4)\) is on the line because \( f(1) = 4 \)

\[ x^2 + y^2 = 1 \]

pick \( x = \frac{1}{2} \)

two outputs fails the VLT.

This is a relation, but not a function.

\[ x^2 + y^2 = 1 \]

\[ y = \pm \sqrt{1 - x^2} \]

two outputs
\[ f(x) = 2x^2 - x + 1 \]

\[ = 2 \left[ \frac{1}{4}x^2 - \frac{1}{8}x + \frac{1}{16} \right] + 1 \]

\[ = 2 \left[ \left( x - \frac{1}{4} \right)^2 - \frac{1}{16} \right] + 1 \]

\[ = 2 \left( x - \frac{1}{4} \right)^2 + \frac{7}{8} \]

**vertex is** \((h, k)\)

**standard form:** \(a(X-h)^2 + k\)

**domain restricted to** \([-1, \infty)\)

\[ f(x) = 2x^2 - x + 1 \]

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
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</thead>
<tbody>
<tr>
<td>-1</td>
<td>2(-1)^2 - (-1) + 1 = 4</td>
</tr>
<tr>
<td>0</td>
<td>2(0)^2 - (0) + 1 = 1</td>
</tr>
<tr>
<td>1</td>
<td>2(1)^2 - (1) + 1 = 2</td>
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</tbody>
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**range:** \([7/8, \infty)\).
\[ f(x) = x^3 - 4x^2 - x + 4 \]
\[ f(x) = 0 \text{ when } x = -1, 1, 4 \]

Intercepts?

Solve \( 0 = 2x^4 + 6x - 7 \) (outputs are zero)

\[
X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
\[
X = \frac{-6 \pm \sqrt{36 - 4(2)(-7)}}{4}
\]
\[
X = \frac{-6 \pm \sqrt{64}}{4}
\]
\[
X = \frac{-6 \pm 8}{4}
\]
\[
X = -1, 1, 4
\]

Intercepts are \((-1, 0), (1, 0), (4, 0)\).

X-coordinate of vertex
Objectives
1. Identify intervals on which a function increases, decreases, or is constant.
2. Use graphs to locate relative maxima or minima.
3. Identify even or odd functions and recognize their symmetries.
4. Understand and use piecewise functions.
5. Find and simplify a function's difference quotient.

If a function changes from increasing to decreasing, there is a local max.
If a function changes from decreasing to increasing, there is a local min.

\[ f(x) = \frac{1}{x^2} \]
Even and odd functions

$y = x^2$

$y = x^3$

$y = x^4$

$y = x^5$

"Even" Functions are symmetrical around the y-axis.
"Odd" Functions have symmetry around the origin.

$y = \frac{1}{x}$

$f(x)$ is odd if

$f(-x) = -f(x)$

$\frac{1}{x} = -\frac{1}{x}$ (The algebra causes the graph to have this symmetry.)

odd odd even

$y = x^4 - x^2 + 1$

$y = x^5 - x^3 + 2$
**Piecewise Functions**

\[ f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \]

**Step Function.**

\[ f(x) = \lfloor x \rfloor = \text{round down to the nearest integer.} \]

- \( f(0.5) = 0 \)
- \( f(-0.5) = -1 \)
- \( f(\pi) = 3 \)
- \( f(e) = 2 \)
- \( f(0.9) = 0 \)
- \( f(0.99) = 0 \)
- \( f(0.9999999999) = 0 \)
Quiz 2

1) Complete the square: \( 2x^2 + 4x - 7 \)

2) Solve and Graph: \( |2x + 1| < 4 \)