Chapter 3: Polynomial and Rational Functions

SECTION 3.1 Quadratic Functions

Objectives
1. Recognize characteristics of parabolas.
2. Graph parabolas.
3. Determine a quadratic function’s minimum or maximum value.
4. Solve problems involving a quadratic function’s minimum or maximum value.

\[ f(x) = ax^2 + bx + c \text{ (general form)} \]

(if \( a > 0 \))

\[ f(x) = a(x - \frac{-b}{2a})^2 + f\left(\frac{-b}{2a}\right) \]

Vertex = \(\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)\)

\[ f(x) = a(x - h)^2 + k \text{ (standard form)} \]

\[ y = a(x - h)^2 + k \]

y = a(x - 3)^2 - 2

Substitute (0, d) into the equation

\[ d = a(0 - 3)^2 - 2 \]

\[ 2 = a \cdot 9 - 2 \]

\[ 4 = a \cdot 9 \]

\[ 4/9 = a \]
general form: \[ y = \frac{4}{9}(x^2 - 3x - 3x + 9) - 2 \]
\[ = \frac{4}{9}(x^2 - 6x + 9) - 2 \]
\[ = \frac{4}{9}x^2 - \frac{8}{3}x + 2 - 2 \]
\[ y = \frac{4}{9}x^2 - \frac{8}{3}x + 2 \]

Graph \( y = -2x^2 + x - 3 \)

4 key points: vertex, y-intercept, x-intercept(s).

Vertex
\[
X = \frac{-b}{2a} = \frac{-1}{2(-2)} = \frac{1}{4} \approx 0.25 \\
y = -\frac{b}{4a} + c = \frac{-1}{8} + \frac{1}{4} - 3 \\
= -\frac{1}{8} + \frac{2}{8} - \frac{24}{8} \\
= -\frac{23}{8} \approx -2.9
\]

y-intercept
\[ y = -3 \]

Other points
\[
\begin{array}{c|c}
X & Y \\
\hline
-1 & -2\cdot(-1)^2 - 6 \\
1 & -2(1)^2 + (1) - 3 = -4
\end{array}
\]

To find the min/max of a parabola determine whether it points up or down and then the y-coordinate of the vertex \((F(\frac{-b}{2a}))\) is the min/max.
You have 200 feet of fencing to enclose a rectangular plot that borders on a river. If you do not fence the side along the river, find the length and width of the plot that will maximize the area. What is the largest area that can be enclosed?

\[ A = w \cdot l \]
\[ = x (200 - 2x) \]
\[ A(x) = -2x^2 + 200x \]

This function points down. The maximum of \( A(x) \) will be

\[ A \left( \frac{-b}{2a} \right) = A \left( \frac{-200}{2(-2)} \right) \]
\[ = A(50) \]
\[ = -2(50)^2 + 200(50) \]
\[ = -2 \cdot 2500 + 10,000 \]
\[ = 5000 + 10000 \]
\[ = 5000 \]

The maximum area is 5000 ft².

width = 50
length = 100

How do we make this a square?

\[ w = x \]
\[ l = 200 - 2x \]

For a square:

\[ w = l \]
\[ x = 200 - 2x \]
\[ 3x = 200 \]
\[ x = \frac{200}{3} \approx 66.7 \]

area would be about 4500 ft².
Rational Functions and Their Graphs

**Objectives**

1. Find the domains of rational functions.
2. Use arrow notation.
3. Identify vertical asymptotes.
4. Identify horizontal asymptotes.
5. Use transformations to graph rational functions.
6. Graph rational functions.
7. Identify slant asymptotes.
8. Solve applied problems involving rational functions.

\[ f(x) = \frac{1}{x} \quad \text{dom}(f) = (-\infty, 0) \cup (0, \infty) \]

\[ g(x) = \frac{2}{x^2 - 10} \]

\[ \text{domain}(g) = (\infty, -\sqrt{10}) \cup (-\sqrt{10}, \sqrt{10}) \cup (\sqrt{10}, \infty) \]

\[ h(x) = \frac{p(x)}{q(x)} \]

Solving \( 0 = q(x) \) gives the values that are not in the domain.

\[ f(x) = \frac{1}{x} \quad \text{what happens when we pick negative values approaching } 0? \]

\[
\begin{align*}
    f(-10) &= -0.1 \\
    f(-2) &= -0.5 \\
    f(-1) &= -1 \\
    f(-0.1) &= -10 \\
    f(-0.01) &= -100 \\
    f(-0.000001) &= -10,000,000
\end{align*}
\]

\[ f(x) \to -\infty \quad \text{as } x \to 0^- \]

We were using numbers from the left.

\[
\begin{align*}
    f(10) &= 0.1 \\
    f(1) &= 1 \\
    f(0.01) &= 100 \\
    f(0.000001) &= 1000000
\end{align*}
\]

\[ f(x) \to \infty \quad \text{as } x \to 0^+ \]