3.3: Division of Polynomials

\[
\frac{17491}{11} = 1590 + \frac{1}{11}
\]

\[
\frac{1590}{11} = 144 + \frac{6}{11}
\]

\[
\frac{64}{6} = 10 + \frac{4}{11}
\]

\[
\frac{55}{5} = 11 + \frac{0}{11}
\]

\[
\frac{99}{9} = 11 + \frac{0}{11}
\]

\[
\frac{0}{0} = 0 + \frac{0}{0}
\]

\[
\frac{3x^3 - 2x + 1}{x} = 3x^2 - 2 + \frac{1}{x}
\]

\[
\frac{3x^2 - 2x + 1}{x} = 3x^2 - 2 + \frac{1}{x}
\]

\[
\frac{2x^2 + 3x - 4}{x + 1} = 2x + 1 + \frac{-5}{x + 1}
\]

\[
\frac{2x^2 + 3x - 4}{x + 1} = 2x + 1 + \frac{-5}{x + 1}
\]

\[
\frac{x^2 - x + 1}{x - 1} = x - 1
\]

\[
\frac{x^2 - x + 1}{x - 1} = x - 1
\]
Objectives
1. Find the domains of rational functions.
2. Use arrow notation.
3. Identify vertical asymptotes.
4. Identify horizontal asymptotes.
5. Use transformations to graph rational functions.
6. Graph rational functions.
7. Identify slant asymptotes.
8. Solve applied problems involving rational functions.

Examples:
\[ y = \frac{x+3}{x^2-1} \]
\[ y = \frac{x^2-x}{x+2} \]
\[ y = \frac{x^3+1}{x^2+1} \]

Graph \( y = \frac{x+3}{x^2-1} = \frac{x+3}{(x+1)(x-1)} + 0 \)

1. Identify asymptotes. \( \pm 1 \) are not in the domain, so we have vertical asymptotes at \( x = 1 \) and \( x = -1 \). Any slant or horizontal asymptote will be equal to the polynomial part. Since the polynomial part is 0, the horizontal asymptote is \( y = 0 \).

2. Identify intercepts:
   - \( y \)-intercept, \( x = 0 \): \[ y = \frac{0+3}{0^2-1} = \frac{3}{-1} = -3 \]
     \[ (0, -3) \]
   - \( x \)-intercept, \( y = 0 \): \[ 0 = \frac{x+3}{x^2-1} \]
     \[ 0 = x + 3 \]
     \[ -3 = x \]
     \[ (-3, 0) \]
3 Graph dotted asymptotes, plot intercepts (graph can't cross vertical asymptotes, but it can cross horizontal asymptotes)

4 Plot some other points

\[
\begin{array}{c|c}
 x & y \\
 \hline
 -\sqrt{2} & \frac{-\sqrt{2} + 3}{(-\sqrt{2})^2 - 1} = \frac{2.5}{-0.75} \approx -3.3 \\
 \sqrt{2} & \frac{\sqrt{2} + 3}{(\sqrt{2})^2 - 1} = \frac{3.5}{-0.75} \approx -4.7 \\
 2 & \frac{2 + 3}{2^2 - 1} = \frac{5}{3} \\
 -2 & \frac{-2 + 3}{(-2)^2 - 1} = \frac{1}{3}
\end{array}
\]
Graph \( y = \frac{x^2 - x}{x + 2} = \frac{x(x-1)}{x+2} = x - 3 + \frac{6}{x+2} \)

<table>
<thead>
<tr>
<th>divide</th>
<th>( \frac{x-3}{x+2} )</th>
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</thead>
<tbody>
<tr>
<td>( x^2 - x + 0 )</td>
<td></td>
</tr>
<tr>
<td>( -x + 2x )</td>
<td></td>
</tr>
<tr>
<td>( -3x + 0 )</td>
<td></td>
</tr>
<tr>
<td>( -3x - 6 )</td>
<td></td>
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<tr>
<td>( 6 )</td>
<td></td>
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<table>
<thead>
<tr>
<th>synthetic division (using linear divisor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2)</td>
</tr>
<tr>
<td>(+)</td>
</tr>
<tr>
<td>(-3)</td>
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</tbody>
</table>

**Asymptotes:** \( X = -2, \quad Y = X - 3 \)

**Intercepts:**
- \( X \)-intercept: \( X = 0 \)
- \( Y \)-intercept: \( Y = \frac{0^2 - 0}{0 + 2} = 0 \)

\( (0,0) \)

\( x \)-intercepts:
- \( 0 = x^2 - x \)
- \( 0 = x(x-1) \)

\( X = 0 \quad X = 1 \)

\( (0,0) \quad (1,0) \)

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3. Graph dotted asymptotes and plot intercepts.
   - Check whether the graph crosses a slant or horizontal asymptote
   - \( y = x - 3 + \frac{6}{x+2} \)
   - \( y = x - 3 \) (no crossing b/c the rational part)

\( X \mid Y = \frac{(x+2)(x-4)}{x+2} = \frac{20}{2} = 10 \) is never 0.
Quiz 3  10/29/13

1. \( f(x) = \sqrt{x+1} \), \( g(x) = x^2 - 5 \). Find the domain of \( f(g(x)) \).

2. Find the distance between \((-2, -3)\) and \((-4, 6)\).