finishing 3.5: graphing with transformations
one applied problem

\[ y = f(x) \quad \text{up by one} \rightarrow y = f(x) + 1 \quad \text{or} \quad y + 1 = f(x) \]

\[ y = f(x) - 2 \quad \text{down by 2} \quad \text{or} \quad y - 2 = f(x) \]

\[ y = f(x + 3) \quad \text{left by 3} \]

\[ y = f(x - 7) \quad \text{right by 7} \]

\[ y = -f(x) \quad \text{flip vertically} \]

\[ y = f(-x) \quad \text{flip horizontally} \]

\[ x = 0 \quad y = \frac{5}{x} \]

\[ x + 5 \]

\[ \frac{2x - 1}{x - 3} = 2 + \frac{5}{x - 3} \]

\[ (x - 3)(2x - 1) = 2x - 6 \]

\[ x = 0 \quad y = \frac{5}{x} \]

\[ x = 2 \quad y = 3 \]

\[ x = 4 \quad y = 7 \]

\[ x = 6 \quad y = 3.7 \]
102. A drug is injected into a patient and the concentration of the drug in the bloodstream is monitored. The drug’s concentration, \( C(t) \), in milligrams per liter, after \( t \) hours is modeled by

\[ C(t) = \frac{5t}{t^2 + 1}. \]

The graph of this rational function, obtained with a graphing utility, is shown in the figure.

a. Use the preceding graph to obtain a reasonable estimate of the drug’s concentration after 3 hours.

b. Use the function’s equation displayed in the voice balloon by the graph to determine the drug’s concentration after 3 hours.

c. Use the function’s equation to find the horizontal asymptote for the graph. Describe what this means about the drug’s concentration in the patient’s bloodstream as time increases.

\[ a. \text{ roughly } 1.5 \text{ mg/L} \quad b. \quad C(3) = \frac{5(3)}{3^2 + 1} = \frac{15}{10} = 1.5 \]

\[ c. \quad C(t) = \frac{5t}{t^2 + 1} = 0 + \frac{5t}{t^2 + 1} \quad \text{horizontal asymptote is } y = 0. \]
Systems of Linear Equations in Two Variables

**Objectives**

1. Decide whether an ordered pair is a solution of a linear system.
2. Solve linear systems by substitution.
3. Solve linear systems by addition.
4. Identify systems that do not have exactly one ordered-pair solution.
5. Solve problems using systems of linear equations.

\[
\begin{align*}
\begin{cases}
y = 2x - 1 \\
y = 3x - 4
\end{cases}
\quad \text{substitution}
\end{align*}
\]

we get to assume that the \( x \)s and \( y \)s in each equation are equal.

\[
2x - 1 = y = 3x - 4
\]

\[
\begin{align*}
2x - 1 &= 3x - 4 \\
-3x &= -5 \\
x &= \frac{5}{3}
\end{align*}
\]

\( y = 3 \left( \frac{5}{3} \right) - 4 = 3(\frac{5}{3}) - 4 = 5 - 4 = 1 \)

\( x = \frac{5}{3}, \quad y = 1 \)

\( (3, 5) \)

\[
\begin{align*}
2x - 4y + 1 &= 0 \\
x + 3y - 1 &= 0
\end{align*}
\]

\[
\begin{align*}
2x - 4y + 1 &= 0 \\
-3y + 1 - 4y + 1 &= 0 \\
-2y + 1 &= 0 \\
y &= \frac{1}{2}
\end{align*}
\]

\[
\begin{align*}
x &= \frac{1}{2} \\
x &= \frac{3}{10} + \frac{10}{10}
\end{align*}
\]

\( x = \frac{1}{10} \)

\( (\frac{1}{10}, \frac{3}{10}) \)
Elimination

\[ \begin{align*}
3x - 2y + 1 &= 0 \\
5x + 2y + 1 &= 0 \\
\hline
8x + 0 + 2 &= 0 \\
-x &= -2 \\
x &= \frac{2}{8} \\
\therefore x &= \frac{1}{4}
\end{align*} \]

\[ \begin{align*}
5x + 2y + 1 &= 0 \\
5\left(\frac{1}{4}\right) + 2y + 1 &= 0 \\
\frac{5}{4} + \frac{8y}{4} &= 0 \\
\frac{5}{4} + 2y &= 0 \\
+ \frac{1}{4} + \frac{1}{4} \\
2y &= \frac{1}{4} \\
\therefore y &= \frac{1}{8}
\end{align*} \]

\( (-\frac{1}{4}, \frac{1}{8}) \)

\[ \begin{align*}
2(4x + 3y + 1) &= 0 \\
2x - 6y + 1 &= 0 \\
\hline
2\left(\frac{-3}{10}\right) + 3y + 1 &= 0 \\
\frac{-6}{5} + \frac{5}{5} - 6y &= 0 \\
\frac{2}{5} - 6y &= 0 \\
\frac{2}{5} &= 6y \\
\frac{2}{30} &= y \\
\therefore y &= \frac{1}{15}
\end{align*} \]

\[ \begin{align*}
8x + 6y + 2 &= 0 \\
2x - 6y + 1 &= 0 \\
\hline
10x + 0 + 3 &= 0 \\
10x &= -3 \\
\therefore x &= -\frac{3}{10}
\end{align*} \]

\( (-\frac{3}{10}, \frac{1}{15}) \)
\[-2(2x-y+3)=0 \quad \text{⇒ same} \quad \Rightarrow \text{infinitely many points of intersection.}\]

\[-4x+2y-6=0\]
\[+ \quad 4x-2y+6=0\]
\[\overline{0=0} \quad \text{true}\]

\[2(3x-y+1)=0 \quad \text{⇒ } 6x-2y+2=0\]
\[-6x+2y+14=0\]
\[+ \quad -6x+2y+14=0\]
\[\overline{0+0+16=0} \quad \text{false}\]

\[No \quad \text{point of intersection}\]