1.1 (still)
examples of finding limits
limits from graphs.

\[ \lim_{x \to 3} 2x^2 - x = 2(3)^2 - (3) = 15 \]

in the limit rules.

\[ \lim_{x \to 2} \frac{x^3 - 4}{x - 2} = \lim_{x \to 2} \frac{(x + 2)(x - 2)}{x - 2} = \lim_{x \to 2} x + 2 = 4 \]

\[ \lim_{x \to 2} \frac{3}{e^{x+1}} = \frac{3}{e^{2+1}} \approx 0.4696 \]

\[ \lim_{x \to 3} \frac{x + 1}{x - 3} = \text{ONE b/c we couldn't cancel the factor } x - 3. \]

\[ \lim_{x \to -0.5} f(x) = 4 \]
\[ \lim_{x \to 3.5^+} f(x) = -1 \]
\[ \lim_{x \to -0.5} f(x) = \text{ONE} \]
\[ \lim_{x \to -1} f(x) = \infty \]
\[ \lim_{x \to -1^+} f(x) = \infty \]
\[ \lim_{x \to 0^+} f(x) = 1 \]
\[ \lim_{x \to 0^-} f(x) = 1 \]
\[ \lim_{x \to 3} f(x) = 1 \neq f(3) \]

jump discontinuity

discontinuity
11.3 Rates of Change

average rate of change of distance function vs. any function

gometric meaning of average rate of change

instantaneous rate of change (next page)

We begin the discussion with a familiar situation. A driver makes the 168-mile trip from Cleveland to Columbus, Ohio, in 3 hours. The following table shows how far the driver has traveled from Cleveland at various times:

<table>
<thead>
<tr>
<th>Time (in hours)</th>
<th>0</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (in miles)</td>
<td>0</td>
<td>22</td>
<td>52</td>
<td>86</td>
<td>118</td>
<td>148</td>
</tr>
</tbody>
</table>

If $f$ is the function whose rule is

$$f(x) = \text{distance from Cleveland at time } x,$$

average speed from $t = 1$ to $t = 2$.

$$\text{average speed } = \frac{\text{total distance}}{\text{total time}} = \frac{118 - 52}{2 - 1} = \frac{66}{1} = 66 \text{ mph}$$

x = during the second hour

slope of the line segment between (1, 52) and (2, 118).

average speed between $t = 0$ and $t = 3$.

$$\text{average speed } = \frac{168 - 0}{3 - 0} = \frac{168}{3} = 56 \text{ mph}$$
26. \( f(x) = x^2 + x \)

**Instantaneous rate of change:** numerically, then algebraically using the limit.

Average rate of change between \( x = 0 \) and \( x = 5 \).

\[
M = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{f(5) - f(0)}{5 - 0} = \frac{30 - 0}{5 - 0} = 6
\]

From \( x = 0 \) to \( x = 4 \):

\[
m = \frac{f(4) - f(0)}{4 - 0} = \frac{20 - 0}{4 - 0} = 5
\]

From \( x = 0 \) to \( x = 3 \):

\[
m = \frac{f(3) - f(0)}{3 - 0} = \frac{12 - 0}{3 - 0} = 4
\]

From \( x = 0 \) to \( x = 2 \):

\[
m = \frac{f(2) - f(0)}{2 - 0} = \frac{6 - 0}{2 - 0} = 3
\]

From \( x = 0 \) to \( x = 1 \):

\[
m = \frac{f(1) - f(0)}{1 - 0} = \frac{2 - 0}{1 - 0} = 2
\]

From \( x = 0 \) to \( x < 0.5 \):

\[
m = \frac{f(0.5) - f(0)}{0.5 - 0} = \frac{0.75 - 0}{0.5 - 0} = 1.5
\]

From \( x = 0 \) to \( x = 0.25 \):

\[
m = \frac{f(0.25) - f(0)}{0.25 - 0} = \frac{(\frac{1}{16} + \frac{1}{4} - 0)}{4} = \frac{0.5}{4} = 0.25
\]

Conclude the instantaneous rate of change of \( f \) at \( x = 0 \) is 1.
The instantaneous rate of change of \( f(x) \) at \( x = a \) is

\[
\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}
\]

(difference quotient)

Find the instantaneous rate of change of \( f(x) = x^2 + x \) at \( x = 0 \)

\[
IRC = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}
\]

\[
= \lim_{h \to 0} \frac{(0+h)^2 + (0+h) - (0^2 + 0)}{h}
\]

\[
= \lim_{h \to 0} \frac{h^2 + h}{h}
\]

\[
= \lim_{h \to 0} \frac{h(h+1)}{h}
\]

\[
= \lim_{h \to 0} h + 1
\]

\[
IRC = 1
\]
Find the instantaneous rate of change of \( g(x) = \sqrt{x-1} \) at \( x = 2 \).

\[
I\!R\!C = \lim_{h \to 0} \frac{g(a+h) - g(a)}{h}
\]

\[
= \lim_{h \to 0} \frac{\sqrt{h+1} - 1}{h} \cdot \frac{\sqrt{h+1} + 1}{\sqrt{h+1} + 1}
\]

\[
= \lim_{h \to 0} \frac{(h+1) - 1}{h(\sqrt{h+1} + 1)}
\]

\[
= \lim_{h \to 0} \frac{h}{h(\sqrt{h+1} + 1)}
\]

\[
= \lim_{h \to 0} \frac{1}{\sqrt{h+1} + 1}
\]

\[
IR\!C = \frac{1}{2} \quad \text{This is like saying that the function has slope } \frac{1}{2} \text{ at } x = 2.
\]

\[
g(2+h) = \sqrt{2+h-1} = \sqrt{h+1}
\]

\[
g(2) = \sqrt{2-1} = 1
\]