Calculus I: Exam 1

No books or notebooks are allowed. Neither are programming and graphing calculators. Please write out your answers in details. I can not give you credit unless you give your intermediate steps and quote corresponding theorems. Please also observe the university policies.

1. (a) For \( f(x) = \frac{a}{x^2} \), where \( a \) is a constant, find and simplify \( f(t+h) - f(t) \).  
   (b) Write the function \( p(x) = \sin \frac{1}{4}(x^2 + 1) \) as a composite function \( g \circ h \).

2. Find the natural domain of the function \( f(x) = \sqrt{\frac{1-\sin^3 x}{\sqrt{|x-1|^2-1}}} \).

3. Sketch the graph of \( f(x) = \frac{|x|}{x} \). Then find each of the following or state that it does not exist.  
   (a) \( \lim_{x \to 0^-} f(x) \),  
   (b) \( \lim_{x \to 0^+} f(x) \),  
   (c) \( \lim_{x \to 0} f(x) \),  
   (d) \( \lim_{x \to -1} f(x) \).

4. Check if the following limits exist; if it exists, find the limit; if not, explain reasons:
   (a) \( \lim_{x \to 3} \frac{x^2-5x+6}{4x^2-36} \);  
   (b) \( \lim_{t \to 0} \frac{\sin(2t)+1-\cos t \sqrt{\sec t + \tan t}}{t} \);  
   (c) \( \lim_{x \to -\infty} \frac{7x^8-x^6+12}{x^8+x^4+100} \);  
   (d) \( \lim_{x \to \infty} x(\sqrt{x^2 + a} - \sqrt{x^2}) \) (Hint \( a^2 - b^2 = (a-b)(a+b) \).)

5. Consider the function \( f(x) \) defined by: \( f(x) = \begin{cases} \frac{1-\cos x}{x}, & \text{if } -1 \leq x < 0; \\ ax + b, & \text{if } 0 \leq x \leq 1; \\ 1, & \text{if } x > 1 \end{cases} \) 
   Determine \( a \) and \( b \) such that \( f(x) \) is continuous on \([-1, 2]\).