Complex II Jan 30

Power Series - absolute convergence, uniform convergence

Thus $\sum A_p z_p (z-w)^p$ converges absolutely about $w$

$z \neq w$ $S$ converges at $z$.

$D \times |x-w| < |z-w|$.

$S$ converges absolutely on $D$ and uniformly on each closed subset of $D$.

Proof (Continued)

$z \in D$

$0 < \delta < |z-w| - |x-w|$

$\alpha = \frac{|x-w| + \delta}{|z-w|} < 1$
\[ m: \text{pos. int.} \quad n \quad \left| \sum_{m+1}^{\infty} A_p \cdot (z-w)^p \right| < \frac{\varepsilon}{2} (1-x) \]

Because \( S \) converges at \( x \) 

\[ q: \text{any pos. int.} \quad \left| A_{m+1} \cdot \left( z-w \right)^{m+1} \right| < \frac{\varepsilon}{2} (1-x) \]

\( t \) be a point \( |x-t| < \delta \)

\[ n: \text{pos. int.} \quad |x-t| + |w-x| < |x-w| + \delta \]

\[ \sum_{m+1}^{\infty} |A_p| \cdot |z-w|^p < \]

\[ \sum_{m+1}^{\infty} |A_p| \cdot (|x-w| + \delta)^p \]

\[ = \sum_{m+1}^{\infty} |A_p| \cdot |z-w|^p \cdot \left( \frac{|x-w| + \delta}{|z-w|} \right)^p \]

\[ < \varepsilon \cdot (1-x) \sum_{m+1}^{\infty} \alpha^p < \varepsilon \]

\[ \sum_{j=0}^{\infty} |x_j|^2 < M \]
\[ \lim_{n \to \infty} \sum_{p=0}^{\infty} A_p (s - w)^p \]

One of the following is true:
1) S is totally divergent, i.e., S converges only at w
2) S is totally convergent, i.e., S converges at every point.
3) S has a radius of convergence, i.e., there is \( r > 0 \) such that S converges at each \( z \) such that \( |z - w| < r \) and S does not converge at any \( z \) such that \( |z - w| \geq r \).
r is the radius of convergence.

Proof let \( B \) be the set of points at which \( S \) converges.

1) \( B \) unbounded

\[ z \in B \quad |z| > |x-w| + |w| + 1 \]

\[ |x-w| < |z| - |w| - 1 < |z-w| \]

2) \( B \) bounded

\[ r = \|z-w\| = \sup \{ |z-w| : z \in B \} \]

a) \( r = 0 \quad \exists w \in S \)

b) \( r > 0 \)

Let \( x : \quad |x-w| < r \)

Let \( z \in B \quad |x-w| < |z-w| \)

Take \( t : \quad |t-w| > r \)

Suppose \( S \) converges at \( t \), i.e., \( t \) is in \( B \)
Then \( r < |t - w| \leq r \)

\[
\begin{align*}
\lim_{x \to \infty} & \quad y'' + y = 0 \\
& \quad x^2 y'' + x(1-x) = 0 \\
y(x) &= \sum A_p x^p \\
S &= \sum A_p (S - w)^p \quad \text{about } w \\
S \text{ number-seq. } \quad S_n = |A_n|^{\frac{1}{n}}
\end{align*}
\]

(i) \( S \) is totally divergent in case the final set of \( s \) is not bounded.

(ii) \( S \) is totally convergent in case \( s \) has limit 0.

(iii) \( S \) has the radius of convergence \( r \) in case the final set of \( s \) is bounded, \( S \) does not have the limit 0, and \( r \) is the greatest number which is a cluster point of \( s \).
\[ \lim_{n \to \infty} \{ z_n \} \quad \varepsilon > 0 \quad N \quad n > N \]

\[ |z_n - L| < \varepsilon \]

\( L \) is the limit of \( z \)

For \( \varepsilon > 0 \) and every \( N \), there is a pos. int. \( n > N \)

\[ |z_n - P| < \varepsilon. \]

\[ \sum_{p=-\infty}^{\infty} A_p \]

\[ p = -2, 1, A_0, A_1, A_2, \ldots \]

\[ \sum A_p (s-\omega)^p \]

\[ \ell^2 \quad ? \quad \frac{1}{s-\omega} \]