**Theorem (Residue Theorem)**

Suppose \( R \) is a simply connected region, and \( \{w_p\} \) is a sequence of points inside \( R \), and \( f \) is a point-finite analytic in \( R - S \). If for each \( p \), \( 1 \leq p \leq n \), \( k_p \) is the residue of \( f \) at \( w_p \), \( K \) is a closed path in \( R - S \), then

\[
\oint_{K} f = \sum_{p=0}^{n} k_p W(K, w_p).
\]
Recall $s_f = s_{\hat{f}}$. 

Proof of $\hat{g}$ is a sequence.

1. \[
\frac{\partial}{\partial \nu} (w_p + \frac{1}{2} - \frac{1}{w_p}) = 0
\]
2. \[
\frac{\partial}{\partial \nu} (w_p + \frac{1}{2} - \frac{1}{w_p}) = 0
\]
3. \[
\frac{\partial}{\partial \nu} \hat{g}(w_p + \frac{1}{2} - \frac{1}{w_p}) = 0
\]
4. \[
\frac{\partial}{\partial \nu} \hat{g}(w_p + \frac{1}{2} - \frac{1}{w_p}) = 0
\]
$$J_F = \sum_{p=0}^{n} - \int \frac{z}{(I-w_p)^2} \frac{w_p + I - w_p}{K}$$

Need Lemma: Using Cauchy Integral Then

For \( z \in \mathbb{R} - K' \), \( h \) analytic

$$\int_{K} \frac{h(z)}{(I-z)^2} = h(z) \int_{K} \frac{1}{I-I}$$

Then

$$\int_{K} \frac{h(z)}{(I-z)^2} = h'(z) \int_{K} \frac{1}{I-I} + h(z) \int_{K} \frac{1}{(I-z)^2}$$

$$= h'(z) 2\pi i \cdot W(K, z)$$

Also if \( y \in \mathbb{R} - K' \)

$$W(y + \frac{1}{I-y} / K, y) = \frac{1}{2\pi i} \int_{K} \frac{1}{y + \frac{1}{I-y} / K}$$

$$= \frac{1}{2\pi i} \int_{K} \frac{1}{y + \frac{1}{I-y} - y} \cdot \frac{1}{(I-y)^2}$$
\[ \frac{-1}{2\pi i} \int \frac{1}{I-y} = -W(K, y) \]

Using these facts,

\[ \int_f = - \sum_{p=0}^{n} \text{Residue at } w_p \cdot 2\pi i \cdot W(w_p + \frac{1}{y'}, K, w_p) \]

\[ = 2\pi i \sum_{p=0}^{n} K_p \cdot W(K, w_p) \]

Def. \( f \) has order \( j \) at \( w \) if \( w \)

is a limit point of the initial set of \( f \), there is a \( V \) such that

\( (*) \) there exist \( r > 0 \) \( i \) \( M > 0 \) such that if \( z \) is in \( J_f \) of \( f \) and \( V \)

\[ 0 < |z-w| < r \]

then \( |f(z)| \leq |z-w|^{-M} \)

and \( j \) is the largest number which

is not less than any such \( V \).
Exer. The point-function $I^{1/2}$ has order $1/2$

at 0. 

$I^{1/2} = E\left(\frac{1}{2} \text{Ln}(1)\right)$

$z = \exp(\frac{1}{2} \text{Ln}(z))$

$\sum \frac{I^w = E(\text{w} \text{Ln})}{|z|^2 = |1z - w|^2}$

$= |z|^2 = |z|^2 1^{1/2}$

$|f(z)| = 1 |z - w|^2 M$

Then if $f$ is analytic in $R$ and

$w$ is a point of $R$, then

(1) if $f$ does not have negative order

at $w$

(2) if $f$ does not have order at $w$, then $f(z) = 0$ for each $z$ in $R$.

(3) if $f$ has order $j$ at $w$, then $j$ is a non-negative integer and there is
A function $g$, analytic in $\mathbb{R}$, such that $g(w) \neq 0$ and $f(z) = (z-w)g(z)$ for each $z$ in $\mathbb{R} - \mathbb{R}^3$.

$$f = \frac{f^{(s)}}{s!}(I-w)^s + \cdots$$

**Coming Attractions**

$$f(z) = g(z) \prod_{0}^{n} (z-w_p)^{j_p}$$

$$\frac{1}{2\pi i} \int \frac{f'}{f} = \sum_{k=0}^{n} j_p \cdot W(K, w_p)$$

$$\left( \ln f \right) = b_{-3} b_{-2} b_{-1} b_0 b_1 - n$$

$$0 0 + \frac{1}{\mathbb{R} - \mathbb{R}^3} (z-w)^{-2} (2-w)^{-n}$$