Isotropic Point Source

Consider isotropic x-ray source, i.e., outgoing flux is same in all directions. $I(r)$ is intensity of flux at distance $r$ from source.

$$I_0 = \int_{x^2+y^2=r^2} I(r) \, ds = 2\pi r I(r)$$

Intensity at distance $r$ from source is $I(r) = I_0/2\pi r$.

$$I(x, y) = I(r) \frac{(x, y)}{\sqrt{x^2 + y^2}} = I_0 \frac{\hat{r}}{2\pi r}$$

If curve $S$ does not enclose source $\int_S I(x, y) \cdot n \, ds = 0$ - with no sources or sinks, what comes in must go out.
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Attenuation of X-rays along line through the source at angle $\phi$ is given by

$$\frac{dI}{dr} = -\left( \mu_a(r, \phi) + \frac{1}{r} \right) I$$

Coefficient is an effective attenuation coefficient due to attenuation by object and beam spreading.

$$\log \frac{I(r_\phi, \phi)}{I_0(r_0, \phi)} = \log \frac{r_0}{r_\phi} - \int_{a_\phi}^{b_\phi} b_{\phi} \mu_a(s, \phi) \, ds$$

Trigonometry gives

$$a_\phi = \frac{l}{\cos \phi} \quad b_\phi = \frac{L}{\cos \phi} \quad r_\phi = \frac{(L + h)}{\cos \phi}$$
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\[ I(r_\phi, \phi) = I_0 \frac{\cos \phi}{2\pi(L + h)} \exp \left[ - \int_{a_\phi}^{b_\phi} \mu_a(s, \phi) \, ds \right] \]

The density of the developed film at a point is proportional to log of total energy incident at that point. To compute energy consider the flux across a part of the film subtended by angle \( \Delta \phi \).

\[ \Delta F = \int_{\phi}^{\phi + \Delta \phi} I(r_\phi, \phi) \hat{r} \cdot n \, d\sigma \]

where \( \hat{r} = -(\sin \phi, \cos \phi) \) is a unit vector, \( n = (0, -1) \) is the outward, unit normal vector to the film plane, and \( d\sigma \) is arc length along film plane.
Point Source Continued

In polar coordinates, \( d\sigma = \frac{L + h}{\cos^2 \phi} d\sigma \). Supposing \( \Delta \phi \) small,

\[
\Delta F \approx \int_{\phi}^{\phi + \Delta \phi} I(r\phi, \phi) \hat{r} \cdot n \frac{L + h}{\cos^2 \phi} d\phi
\]

\[
\approx I_0 \frac{\cos^2 \phi}{2\pi(L + h)} \exp \left[ - \int_{a\phi}^{b\phi} b(\mu_a(s, \phi)) \frac{L + h}{\cos^2 \phi} \Delta \phi \right.
\]

Length of film subtended by angle \( \Delta \phi \) is approximately \( \Delta \sigma = \frac{L + h}{\cos^2 \phi} \Delta \phi \), so that

\[
\frac{dF}{d\sigma} = \frac{I_0 \cos^2 \phi}{2\pi(L + h)} \exp \left[ - \int_{a\phi}^{b\phi} b(\mu_a(s, \phi)) ds \right]
\]