Assignment 1 Problem Solving Seminar, Spring 06

Work as many parts of these problems as you can - the assignment is due on Thursday evening, February 2, at the beginning of class. The problems are sequential, but if you get stuck on one part, you jump ahead and try other parts. As always, there may be typing errors - part of your assignment is to find them and make sense of the problem.

Defining surfaces via functions of two variable or equations. Consider the following two-dimensional surfaces in \( \mathbb{R}^3 \).

1. Find an implicit representation for a plane, sometimes called a hyperplane, of the form \( \{(x, y, z) : \} \). In its most general form this will involve 4 parameters. Then, assuming the plane is not parallel to the \( z \) axis, find an explicit representation of the plane, of the form \( g(x, y) = \).

2. Now consider a sphere. For simplicity, make the origin the center and radius 1. This unit sphere is two-dimensional, but lives in \( \mathbb{R}^3 \). The standard notation is \( S^2 \). (\( S^1 \) is the unit circle and so on). Once again find an implicit representation for \( S^2 \): the set of all \( (x, y, z) \) such that... Find an explicit representation as \( g(x, y) = \) - at least for the upper hemisphere.

3. Research spherical coordinates - a generalization of polar coordinates. What is the equation for \( S^2 \) in this coordinate system? Would the equation for a plane be easier or harder to write down using spherical coordinates?

4. (Harder) Suppose you take the upper hemi-torus (the top part of a bagel cut in half). Now you have an inner an outer radius, \( 0 < R_1 < R_2 \). Can you find either a function or an equation for representing this manifold? Gluing looks much easier?

5. Starting from the other direction, consider the function \( g(x, y) = ax^2 + 2bxy + cy^2 \), what does the manifold look like for different choices of \( a, b, c \)? Take two specific cases: (1) \( a = 1, b = 0, c = 1 \) and (2) \( a = 1, b = 0, c = -1 \). If you have access to a plotting program great, otherwise plot a few points and look at the image under \( g \) of the \( x \)-axis and the \( y \)-axis.